Chapter 4
Systems of Linear Equations and Inequalities

Exercise Set 4.1

2. The lines intersect at one point (the solution).
4. The lines are identical.
6. Eliminate $y$, because $2y$ and $-2y$ are additive inverses.
8. Yes
10. No
12. Yes
14. No
16. \[
\begin{align*}
x - y &= 8 \\
x + y &= 12
\end{align*}
\]
18. \[
\begin{align*}
2l + 2w &= 65 \\
l &= w + 5
\end{align*}
\]
20. \[
\begin{align*}
2x + y &= 8 \\
x - y &= 1
\end{align*}
\]

(3, 2)

22. \[
\begin{align*}
y &= -2x - 1 \\
y &= x - 1
\end{align*}
\]

(-1, -2)

24. \[
\begin{align*}
3x + 2y &= 6 \\
6x + 4y &= 18
\end{align*}
\]
no solution.

26. \[
\begin{align*}
2x + y &= -2 \\
x &= -3
\end{align*}
\]
all ordered pairs that solve $2x + y = -2$

28. \[
\begin{align*}
y &= x - 3 \\
(-3, 4)
\end{align*}
\]

30. a) consistent with independent equations
b) one solution
32. a) inconsistent
b) no solution
34. a) consistent with dependent equations
b) infinite number of solutions
36. consistent with independent equations
38. consistent with dependent equations
40. inconsistent
42. (1, -3)
44. (2, -4)
46. \[
\begin{pmatrix}
1 \\
2 \\
1 \\
3
\end{pmatrix}
\]
48. (2, 3)
50. (-2, -4)
52. (4, -2)
54. no solution
56. all of the ordered pairs along $x + y = -5$.
58. Mistake: Substituted into the same equation.
Correct: (5, -1)
60. (6, -1)
62. (4, -3)
64. (2, 2)
66. \( \left( 2, -\frac{1}{4} \right) \)

68. \((-2, 5)\)

70. \(\begin{pmatrix} 24 \\ 22 \\ 19 \end{pmatrix} \)

72. \((3, 2)\)

74. \((-3, 3)\)

76. no solution

78. all of the ordered pairs along the line
\[ 4x + 2y = 6 \]

80. Mistake: Multiplied only the left side of the second equation by \(-1\).
Correct: \((2, 1)\)

82. a) 4000  
   b) \$500  
   c) \(x > 4000\)

84. a) \[ \begin{align*}
   c &= 22,500 \\
   c &= 350n + 1500
\end{align*} \]

b) \[ \begin{align*}
   c &= 350n + 1500 \\
   c &= 22,500
\end{align*} \]

Exercise Set 4.2

2. We eliminate the same variable from two pairs of equations to get two equations with the same two variables.

4. After adding two equations, if the resulting equation has no variables and is false, then the system has no solution.

6. It would be on the line of intersection of two planes but not on the third plane.

8. Yes

10. No

12. Yes

14. \((1, -2, 3)\)

16. \((4, 0, -3)\)

18. \((4, -2, 3)\)

20. no solution (inconsistent)

22. infinite number of solutions (dependent equations)

24. \((-2, 1, -3)\)

26. \(\left( \frac{1}{2}, -1, \frac{1}{4} \right) \)

28. \((3, 1, -2)\)

30. \((2, -1, 2)\)

32. \((-3, 7, 4)\)

34. \((-3, 2, -3)\)

Exercise Set 4.3

2. With the wind: \(x + y\)
Against the wind: \(x - y\)

4. \[ l = (s + m) - 0.2 \]

6. House: \$77 million; Senate: \$20 million.

8. 496 thousand

10. 2007: \(145,913\) thousand; 2008: \(146,046\) thousand


14. 23 large candles and 50 small candles

16. 4.9%

18. Width is 36 ft. and the length is 78 ft.

20. 55º and 35º

22. 240 seconds (4 minutes)
24. 11:30 A.M.
26. boat: 21 mph; current: 3 mph
28. plane: 300 mph; wind: 20 mph
30. 12 L of 45% and 8 L of 30%
32. $8000 at 7% and $4000 at 4%
34. 4, 5, and 9
36. 8 in., 10 in., and 15 in.
38. Pens are $2.00, erasers are $0.50, and paper is $1.00.
40. 80 child, 115 student, and 105 adult
42. 5 first-place, 3 second-place, and 6 third-place
44. 120 lbs. of peanuts, 60 lbs. of almonds, and 20 lbs. of pecans
46. 8 lbs. of Jamaican, 7 lbs. of Hawaiian, and 10 lbs. of Sulawesi
48. money market: $1666.67; income fund: $500; growth fund: $2833.33
50. concert: 120 dB; normal speaking: 60 dB; jet: 130 dB
52. \( a = -16, v_0 = 50, h_0 = 100; \\ h = -16r^2 + 50r + 100 \)

Puzzle Problem
45, 47, 64, 72

Exercise Set 4.4

2. Arrange each equation so that all variables are in the same order on the left side of the equal sign and the constant term is on the right side of the equal sign. Omit all of the variables and the equal signs. Place a vertical dashed line where the equal signs were so that it is between the coefficients of the variables and the constants.

4. The rules are the same for both processes. With matrices, we use only the coefficients and the constants. With the elimination method, we also use the variables and the equal sign.

6. The bottom equation represents the value of the last variable. Substitute the known value into the equation above to determine the value of the next variable. Continue until all values for the variables are found.

8. \( \begin{bmatrix} 5 & 6 & | & 2 \\ 10 & 3 & | & -2 \end{bmatrix} \)
10. \( \begin{bmatrix} 3 & 0 & | & 6 \\ 9 & 2 & | & -2 \end{bmatrix} \)
12. \( \begin{bmatrix} 3 & 2 & -3 & | & 2 \\ 2 & -4 & 5 & | & -10 \end{bmatrix} \)
14. \( \begin{bmatrix} 1 & -3 & 0 & | & 3 \\ -3 & 4 & 9 & | & 3 \end{bmatrix} \)
16. \( (-8,-5) \)
18. \( (-4,-1,3) \)
20. \( \begin{bmatrix} 1 & 2 & | & -2 \\ 0 & 2 & | & 2 \end{bmatrix} \)
22. \( \begin{bmatrix} 1 & 5 & -3 & | & 8 \\ 0 & 17 & -13 & | & 18 \end{bmatrix} \)
24. \( \begin{bmatrix} 0 & 1 & -2 & | & -6 \\ 0 & 0 & 1 & | & -4 \end{bmatrix} \)
26. Replace \( R_2 \) with \(-5R_1 + R_2\).
28. Replace \( R_1 \) with \(5R_2 + R_3\).
30. \( (-3,5) \)
32. \( (3,-4) \)
34. \( (-1,4) \)
36. \( (6,0) \)
38. \( (0,0) \)
40. \( (2,2) \)
42. \( (2,-1,2) \)
44. \( (-1,3,2) \)
46. \( (-3,-3,3) \)
48. \( (-2,-3,4) \)
50. \( (-1,7,-3) \)
52. \( (-9,6,-2) \)
54. \( (8,7) \)
56. \( (8,11) \)
58. \( (4,8,-10) \)
60. \( (-5,8,12) \)
62. Mistake: In the matrix being created, the coefficient 4 was placed in the x-position instead of the y-position.
Correct: The original matrix should be
\[
\begin{bmatrix}
1 & -1 & 1 & 8 \\
3 & 0 & -1 & -9 \\
0 & 4 & 1 & -6
\end{bmatrix}
\]. The solution is \((-1, -3, 6, 1)\).

64. General admission: $15; student: $5
66. Saint Gotthard Tunnel: 10.1 miles; Arlberg Tunnel: 8.7 miles
68. money market: $38,000; growth fund: $22,000
70. Salmon is $8; tuna is $7; cod is $5
72. 0 three-point field goals; 5 two-point field goals; 7 free throws
74. \(F_1 = 100, F_2 = 250, F_3 = 500\)

**Exercise Set 4.5**

2. A matrix is a rectangular arrangement of numbers. A determinant is a single-value evaluation of a square matrix.
4. In the coefficient matrix, replace the y-column values with the constant-column values and evaluate the determinant of the resulting matrix.
6. 25
8. \(-38\)
10. 28
12. 2
14. 30
16. \(-10\)
18. 3
20. 20
22. 16
24. 16
26. 6.28
28. \(-12.232\)
30. \(\frac{3}{5}\)
32. \(\frac{-13,823}{7200}\)
34. \(4x + 18y + 12\)
36. \((-5, 2)\)
38. \((-6, 1)\)
40. \((3, -3)\)
42. no solution
44. \((3, -4)\)
46. \((3, -6)\)
48. \((-3, -2)\)
50. \((3, -1, 2)\)
52. \((-4, 3, -1)\)
54. \((-1, -1, 1)\)
56. \(\left(\frac{3}{4}, \frac{1}{2}, 5\right)\)
58. infinite solutions
60. \((3, 2, 1)\)
62. \(x = -7\)
64. \(x = 3\)
66. 14.5 square units
68. 20 square units
70. 12.5 square units
72. 27.7% on taxes; 15.6% on housing
74. 8 inches, 8 inches, and 5 inches
76. 5 three-point shots; 23 two-point shots; 28 free throws

**Exercise Set 4.6**

2. The solution set for the system contains all ordered pairs in the region where the solution sets overlap along with all ordered pairs on the portion of any solid line that touches the region of overlap.
4. The solution region is the entire first quadrant.
6. \[
\begin{align*}
x & > 0 \\
y & < 0
\end{align*}
\]
8. no solution

10.

12.

14.

16.

18.

20.

22.

24. no solution

26.

28.

30.

32.
34. The boundary line for $x - 3y \geq 2$ should be a solid line.

36. The boundary line for $x + 2y \geq 5$ is graphed incorrectly.

40. The boundary line for $x - 3y \geq 2$ should be a solid line.

42. The boundary line for $x + 2y \geq 5$ is graphed incorrectly.

44. a) \[
\begin{align*}
  l \geq w + 25 \\
  2w + 2l \leq 500
\end{align*}
\]
b) \[
\begin{align*}
  l \geq w + 25 \\
  2l + 2w \leq 500
\end{align*}
\]
c) Answers may vary. Two examples are (100, 50) and (200, 25).

46. a) \[
\begin{align*}
  V + M \geq 1150 \\
  V \geq 550
\end{align*}
\]
b) \[
\begin{align*}
  V + M \geq 1150 \\
  V \geq 550 \\
  V \leq 800 \\
  M \leq 800
\end{align*}
\]
c) \[
\begin{align*}
  V + M \geq 1150 \\
  V \geq 550 \\
  V \leq 800 \\
  M \leq 800
\end{align*}
\]
d) Answers may vary. Two examples are (600, 700) and (750, 650).

**Collaborative Exercises**

2. $x \geq 0; y \geq 0$

4. $0, 0, 16, 4, 7, 13, 0, 16.5, 18, 0$

5. $350x + 215y$

6. The point (16, 4) yields a maximum profit of $6460$. 