Inverse functions:

Let $f$ and $g$ be two functions

\[ f(g(x)) = x \quad \forall \ x \text{ in the domain of } g \]

AND

\[ g(f(x)) = x \quad \forall \ x \text{ in the domain of } f \]

The function $g$ is the inverse of the function $f$ and is denoted by $f^{-1}$ (read "$f$ inverse").

Thus,

\[ f(f^{-1}(x)) = x \quad \text{AND} \quad f^{-1}(f(x)) = x \]

The domain of $f$ is equal to the range of $f^{-1}$, and vice versa.

To show that $f(x)$ and $g(x)$ are inverses we "verify" that $f(g(x)) = x$ AND $g(f(x)) = x$.

Finding the inverse of a function:

1. Replace $f(x)$ with $y$

2. Interchange $x$ and $y$

3. Solve for $y$. If this equation does not define $y$ as a function of $x$, then $f$ has no inverse function.

4. If this inverse IS a function, replace $y$ with $f^{-1}(x)$

To verify, show that $f(f^{-1}(x)) = x$ AND $f^{-1}(f(x)) = x$