Commutative Property of Addition

This property involves 2 numbers, variables, or expressions. It simply says \( 2 + 3 \) equals \( 3 + 2 \) (the order has changed). In the grocery store, it doesn't matter if your bread or milk goes first on the conveyor belt for the cashier to ring up. You pay the same price either way.

The order is important for subtraction. The answer is different.

\[
8 - 4 \text{ is not the same as } 4 - 8
\]

Commutative Property of Multiplication

This property involves 2 numbers, variables, or expressions. It simply says \( 2 \times 3 \) equals \( 3 \times 2 \) (the order has changed). In the office supply store, the total price of 5 pens that are $2 each is the same as 2 pens that are $5 each.

The order is important for division. The answer is different.

\[
\frac{8}{4} \text{ is not the same as } \frac{4}{8}
\]

Associative Property of Addition

This property involves 3 or more numbers, variables, or expressions. The grouping of addition doesn't matter.

\[
(3 + 4) + 5 \text{ equals } 3 + (4 + 5) \quad \text{and} \quad 3 + 4 \text{ are added first, } \quad 4 + 5 \text{ are added first}
\]

In the grocery store, it doesn't matter if the apples and oranges are on the conveyor belt first or if the oranges and lettuce are first. The total price for the apples, oranges, and lettuce will be the same.

The grouping for subtraction is important. The answer is different.

\[
(4 - 1) - 3 \quad \text{is not equal to} \quad 4 - (1 - 3)
\]

\[
3 - 3 \quad \# \quad 4 - (-2)
\]

\[
0 \quad \# \quad 4 + (+2)
\]

\[
0 \quad \# \quad 6
\]
1.10 – 2 PROPERTIES OF THE REAL NUMBER SYSTEM

Examples and Explanations by Linda Shirk – To accompany MCC’s textbook for MATH 0930/0931/0960

Associative Property of Multiplication

This property involves 3 or more numbers, variables, or expressions. The grouping of multiplication doesn’t matter.

\[(5 \text{ days} \times 8 \text{ hours}) \times \$10 \text{ equals } 5 \text{ days} \times (8 \text{ hours} \times \$10)\]

\[
\uparrow \\
5 \text{ and } 8 \text{ are multiplied first} \\
\text{and } 8 \text{ and } 10 \text{ are multiplied first}
\]

The grouping for division is important. The answer is different.

\[(8 ÷ 4) ÷ 2 \quad \text{is not equal to} \quad 8 ÷ (4 ÷ 2)\]

\[
\begin{align*}
2 ÷ 2 & \neq 8 ÷ 2 \\
1 \neq 4
\end{align*}
\]

Distributive Property

\[a \times (b + c) \text{ equals } a \times b + a \times c \quad \text{(or } ab + ac)\]

\[\text{OR}\]

\[a \times (b - c) \text{ equals } a \times b - a \times c \quad \text{(or } ab - ac)\]

The distributive property allows for “getting around” the requirement of doing operations in parentheses first in Order of Operations (PEMDAS).

At a 50\% off sale, a cashier could add all the purchases and then take the 50\% discount (multiply by 0.5 or divide by 2).

OR a cashier could take the 50\% discount on every purchase (multiply by 0.5 or divide by 2) and add the sale prices. The total cost is the same either way.

Example

If \(a = 2\) and \(b = 3\) and \(c = 4\)

\[a \times (b + c) = a \times b + a \times c\]

\[2 \times (3+4) = (2 \times 3) + (2 \times 4)\]

\[
\uparrow \\
\text{add first} \\
\uparrow \\
\text{multiply first, multiply second} \\
\uparrow \\
\text{then multiply} \\
\uparrow \\
\text{then add}
\]

\[2 \times (7) = (6) + (8)\]

\[
\begin{align*}
14 & = 14
\end{align*}
\]
The distributive property also applies if 3 or more terms are inside the parentheses.

\[ 2 \times (3 + 4 + 5) \text{ equals } 2 \times 3 + 2 \times 4 + 2 \times 5 \]

The number on the outside of the parentheses is "distributed" to each number inside of the parentheses.

\[ 2 \times (3 + 4 + 5) \]

Identity Properties

\[ 5 + 0 \text{ equals } 5 \]
Adding 0 doesn't change numbers.
0 is the additive identity.

\[ 5 \times 1 \text{ equals } 5 \]
Multiplying by 1 doesn't change numbers.
1 is the multiplicative identity.

Inverse Properties

\[ 5 + (-5) \text{ equals } 0 \]
Adding the opposite of a number gives 0.
-5 is the additive inverse for 5.

\[ 5 \times \frac{1}{5} \text{ equals } 1 \]
Multiplying by the reciprocal gives 1.
\( \frac{1}{5} \) is the multiplicative inverse (or reciprocal) for 5.

This can't be 0.
Zero doesn't have a reciprocal.

Anything \( \times 0 \) equals 0.

Anything \( \times 1 \) equals the original thing.