These are a second type of problems solved using a table. The explanation will be less detailed than before. For more details about these problems, go back to the first motion example.

There are more formulas (equations) to choose from to use in the top row of the table. But in writing the equation to solve, mostly ADDITION will be used. Some of the problems will involve interest (I=PRT) and an extra column will be used in these tables. And a final change is, in most problems, a total number will be known, but the parts won’t be, so setting up the variable expressions can be tricky.

Below are the formulas which will be used:

**Interest**

Principal \( \times \) Rate \( \times \) Time = Interest

\[
\text{\$} \times \frac{\%}{\text{time}} \times \text{time} = \text{\$}
\]

**Monthly Payments** (cable, telephone, rent)

Rate \( \times \) Time = Amount Paid

\[
\frac{\$}{\text{month}} \times \text{months} = \text{\$ paid}
\]

**Pay** (hourly, weekly, monthly)

Rate \( \times \) Time = Amount Paid

\[
\frac{\$}{(hr)(wk)(mo)} \times (hr)(wk)(mo) = \text{\$ paid}
\]

An alternate method to using these formulas is to use a generic formula which is easier to remember for tests. The formula is **RATE \( \times \) NUMBER = AMOUNT**
The first type of money problem involves investing money in 2 different investments that earn different rates of interest. Investments that are more "risky" (subject to losing money) pay more in interest to compensate for the "riskiness." The formula used is

\[ \text{INTEREST} = \text{PRINCIPAL} \times \text{RATE} \times \text{TIME} \]

(Money earned) (Money invested) (Percent) (Years, usually)

The "small" number The "big" number

The textbook shows on p. 214 that the interest amounts can be added, subtracted, or set equal to each other. To make these problems easier to do, I will only show ones where ADDITION is used.

These problems will give a total number that will need to be split up, using the variable. This process will be explained in detail in the first example, but not in the later ones.

p. 204 Ex 3

Carmine will invest a total of $15,000 in 2 investments.

A loan will pay him 8% simple interest (compound interest is complicated).

A one-year certificate of deposit will pay him 5%.

He wants to earn a total of $1,125 interest from both. How much should he put in each investment?

CD = 5% for 1 year
Loan = 8% for 1 year

Put these in the table.

<table>
<thead>
<tr>
<th>Principal x Rate x Time</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>.05 1</td>
</tr>
<tr>
<td>loan</td>
<td>.08 1</td>
</tr>
</tbody>
</table>

Those were the easy parts.

Now to do the harder part, the variable.
The total amount of money (principal) which will be split up into 2 parts is $15,000. The question to answer here is, “How much goes into each type of investment?” To answer this, we’ll temporarily use 2 variables for the 2 investments.

Let \( x \) = amount invested in the CD

Let \( y \) = amount invested in the loan

\[
\text{Amount in CD} + \text{Amount in loan} = \$15,000
\]

\[
x + y = 15,000
\]

Unfortunately, we don’t get to use 2 variables in this class; we only get to use one. So use this equation and find out what \( y \) is:

\[
x + y = 15,000 \quad \text{(Solve for } y\text{)}
\]

\[
x + y = 15,000
\]

\[-x\]

\[
y = 15,000 - x
\]

So we won’t use \( y \) in the table; we’ll use \( x \) and \( 15,000 - x \)

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Time</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>( x )</td>
<td>.05</td>
<td>1</td>
</tr>
<tr>
<td>loan</td>
<td>( 15,000 - x )</td>
<td>.08</td>
<td>1</td>
</tr>
</tbody>
</table>

\( x \) = amount invested in CD

\( 15,000 - x \) = amount invested in loan

Be careful. Many students mistakenly use \( x \) and \( x - 15,000 \). The 2 variable expressions must add up to the total \( 15,000 \).

Wrong

\[
x + (x - 15,000)
\]

\[
2x - 15,000
\]

Right

\[
x + (15,000 - x)
\]

\[
15,000
\]
The boxes are filled. It's time to multiply them and write the answer in the right box labeled "Interest."

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Time</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>x</td>
<td>.05</td>
<td>x .05</td>
</tr>
<tr>
<td>loan</td>
<td>15,000 - x</td>
<td>.08</td>
<td>(15,000 - x) .08</td>
</tr>
</tbody>
</table>

Neither box looks very "algebraic."

\[ \text{Make sure this is distributive with parentheses.} \]

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Time</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>x</td>
<td>.05</td>
<td>.05x</td>
</tr>
<tr>
<td>loan</td>
<td>15,000 - x</td>
<td>.08</td>
<td>.08(15,000 - x)</td>
</tr>
</tbody>
</table>

You can write them so they do look more "algebraic." The changes occurred in the right column.

The total interest from both investments is to be $1,500.

The equation looks like this in words:

\[
\text{CD interest} + \text{loan interest} = 1125
\]

Substitute:

\[
.05x + .08(15,000 - x) = 1125
\]

\[
.05x + 1200 - .08x = 1125
\]

Combine like terms:

\[
-.03x + 1200 = 1125
\]

Negatives are OK:

\[
\frac{-1200}{-.03x} = -75
\]

\[
\frac{-.03x = -75}{-.03} = 2500
\]

So $2,500 would be invested in the CD.

The rest, $15,000 - x ($15,000 - 2,500) or $12,500 would be invested in the loan.
Why did we choose the CD to be represented by \( x \)? It was arbitrary. We could just as easily let the loan be represented by \( x \). If we had done that, the first answer found, \( x \), would have been 912,500. It also would have given an equation without negative values. Either way is correct. The solution book will only show one of the 2 methods.

Here is the other method with less detail in solving the equation.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Time</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD 15,000 - ( x )</td>
<td>.05</td>
<td>1</td>
<td>.05(15,000 - ( x ))</td>
</tr>
<tr>
<td>loan ( x )</td>
<td>.08</td>
<td>1</td>
<td>.08( x )</td>
</tr>
</tbody>
</table>

\[
.05(15,000 - x) + .08x = 1125 \\
750 - .05x + .08x = 1125 \\
750 + .03x = 1125 \\
750 \\
.03x = 375 \\
.03 \\
\frac{375}{.03} = 12,500
\]
The book doesn’t have any examples like this one. This is #39 on page 213.

Extra Example:

There was a telephone rate increase during the year (Jan - Dec). The old rate was $17.10. The new rate is $15.40. The total telephone expense for the year was $207.80. During which month did the increase go into effect?

A new formula for the table is needed. They’re on page 13. The most appropriate one is “Monthly Payments”:

\[ \text{Rate} \times \text{Time} = \text{Amount Paid} \]

Set up the table.

<table>
<thead>
<tr>
<th>Rate \times Time</th>
<th>Amount Paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old 17.10</td>
<td></td>
</tr>
<tr>
<td>New 15.40</td>
<td></td>
</tr>
</tbody>
</table>

We’re looking for a month, but we’ll have to do it with numbers. A year has 12 months, so the total number of telephone payments (the total “times” a payment was made) is 12. The 2 variable expressions in the “time” column must add up to 12. Again, we only get one variable. One expression will be \( x \). The other variable expression will be \( 12 - x \).

\[ x = \text{no. of payments at old rate} \]
\[ 12 - x = \text{no. of payments at new rate} \]

These were chosen arbitrarily.
Extra Example-cont.

The table columns to the left are filled in. Now use the formula to multiply and fill in the right column.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Amount Paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old</td>
<td>17.10</td>
<td>x</td>
</tr>
<tr>
<td>New</td>
<td>15.40</td>
<td>12 - x</td>
</tr>
</tbody>
</table>

The total amount paid was $207.80.

The equation in words: Then substitute from the table.

\[
\text{Amount Paid at Old Rate} + \text{Amount Paid at New Rate} = \text{Total Paid}
\]

\[
17.10x + 15.40(12 - x) = 207.80
\]

Combine like terms

\[
17.10x + 220.80 - 15.40x = 207.80
\]

\[
17.0x + 220.80 = 207.80
\]

\[
17.0x + 220.80 = 207.80
\]

\[
\frac{220.80}{-13.00} = -13.00
\]

\[
\frac{-1.30x}{-1.30} = 10
\]

So the old rate was paid for 10 months. The new rate was paid for 2 months. The 11th month is November.
p. 205 Ex 4
Again, there are 2 ways to set up this problem. Here, it will be done differently from the way in the book.
- Single rockers - $130
- Double rockers - $240
- Total sold - 10
- Total money for rockers - $1740. Find the number of single and double rockers sold.
The formula to use is "Buying or Selling by the Item" (see them on p. 12)
Rate \times Number = Amount Paid - write it in the top line of the table.

<table>
<thead>
<tr>
<th>Rate x</th>
<th>Number = Amount Paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 130</td>
<td></td>
</tr>
<tr>
<td>D 240</td>
<td></td>
</tr>
</tbody>
</table>

Also fill in the prices for single and double

The variable expressions will be in the "Number" column. The total number of rockers is 10, so the variable expressions will be:

\[
x \quad \quad \quad \quad 10 - x
\]
They must add up to 10.

\[
x + (10-x) = x + 10 - x = 10
\]
Here, \(x\) will represent the double rockers (which is different from the book).
So the single rockers will be 10 - \(x\).

<table>
<thead>
<tr>
<th>Rate x</th>
<th>Number = Amount Paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 130</td>
<td>10 - x</td>
</tr>
<tr>
<td>D 240</td>
<td>(x)</td>
</tr>
</tbody>
</table>

Do what the formula in the top line says - Multiply 'Rate' times 'Number' and write the result in the last box.

<table>
<thead>
<tr>
<th>Rate x</th>
<th>Number = Amount Paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 130</td>
<td>10 - x 130 (10 - x)</td>
</tr>
<tr>
<td>D 240</td>
<td>(x) 240 (x)</td>
</tr>
</tbody>
</table>
Now the equation can be written from the last column and the total amount of money.

In words, the equation looks like this:

Amount from single rockers + Amount from double rockers = Amount Paid

Substitute: \[ 130(10-x) + 240x = 1740 \]

Solve the equation:

\[
\begin{align*}
130(10-x) + 240x &= 1740 \\
1300 - 130x + 240x &= 1740 \\
-1300 + 110x &= 1740 \\
110x &= 440 \\
x &= 4 \\
\end{align*}
\]

\[ x = 4 \]  This answer must be a whole number.

So 4 double rockers were sold.

\[ 10 - x = \text{single rockers sold} \]

\[ 10 - 4 = \]

6 = single rockers sold

Check: \[
\begin{align*}
$130 \times 6 &= $780 \\
$240 \times 4 &= $960 \\
$780 + $960 &= $1740 \text{ Correct.}
\end{align*}
\]