Plotting Points - This is a skill you will need throughout all the algebra classes and even into calculus if you go that far. To graph a line, only 2 points are needed, but we will find 3 points and the third point will be a "check" on the work. These are the steps:

1. Solve the linear equation for $y$ if it isn't already. This step is optional, but does make the process easier.
2. Choose values for $x$, substitute them in the equation and solve for the corresponding $y$ value. This process is used throughout algebra and calculus.*
3. Plot the points found in step 2.
4. Connect the points to form a line (hopefully, the "line" is straight.)*

* The one value for $x$ that should always be used is 0. Zero being multiplied in terms will make them 0 and the calculations will be easier. Putting the values in table form makes the plotting easier.

$p 412 Ex 1$
Graph $y = 2x + 4$

Make a Table

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Do substitutions

$y = 2x + 4$ $x = 0$
$y = 2(0) + 4$
y = 4

You could put your finger over the $2x$ term and "make it disappear" and see that $y = 4$. 4
Ex 1 - cont.

The points appear to be co-linear and can be connected with a line. If the value 2 were chosen for \( x \), the corresponding value of \( y \) would be 12, and off the graph. So, sometimes other values for \( x \) need to be chosen. Yes, choosing positive values for \( x \) might make the math easier, but (as before) might give values off the graph. The values of \( x \) should be around zero to help prevent this.

**p.432** At the end of example 1 is an important sentence: The graph of an equation is a picture of the set of points whose coordinates satisfy the equation (make the equation a TRUE statement).

**p.412** Ex 2

Graph \( 3y = 5x - 6 \) The work could be done with the equation in this form, but it is much easier if it is solved for \( y \).

\[
\frac{3y}{3} = \frac{5x - 6}{3} \\
y = \frac{5}{3}x - \frac{6}{3} \\
y = \frac{5}{3}x - 2
\]

Values for \( x \) need to be chosen. Zero is the best and easiest. Again, just putting your finger over \( \frac{5}{3}x \) shows that \( y = -2 \).

If \( x = 1 \) is chosen, this is what results:

\[
\begin{align*}
y &= \frac{5}{3}x - 2 \\
y &= \frac{5}{3}(1) - 2 \\
y &= \frac{5}{3} - 2 \\
y &= 1 \frac{2}{3} - 2 \\
y &= -\frac{1}{3}
\end{align*}
\]

Messy and not fun! \((1, -\frac{1}{3})\) would be difficult to graph (plot) because \(-\frac{1}{3}\) would have to be estimated. This can be avoided by choosing \( x \)-values that are multiples of the denominator with \( x \). In this example, the \( x \)-term has a coefficient of \( \frac{5}{3} \), so choose multiples of 3.
Ex 2 - cont.
Graph \( y = \frac{5}{3}x - 2 \)

Make table

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>-3</td>
<td>-7</td>
</tr>
</tbody>
</table>

Choose values of \( x \)

Find values of \( y \)
\[
\begin{align*}
  y &= \frac{5}{3}x - 2 \quad x = 0 \\
  y &= \frac{5}{3}x - 2 \quad x = 3 \\
  y &= \frac{5}{3}x - 2 \quad x = -3 \\
  y &= \frac{5}{3}(3) - 2 \\
  y &= \frac{5}{3} \cdot \frac{-1}{3} - 2 \\
  y &= \frac{5}{3}(-1) - 2 \\
  y &= -5 - 2 \\
  y &= -7 
\end{align*}
\]

Put \( y \) values in table

Graph the points from this table. Connect the points to make a line. Arrows go on both ends to show that the line extends in both directions infinitely.

p. 413 Ex. 3

This example shows that any line equation with only \( x \) and \( y \) terms and no number term goes through the origin. When the equation is solved for \( y \) the result is: \( y = -\frac{5}{3}x \). When \( 0 \) (zero) is chosen as the value for \( x \) and substituted in the equation, the result is that \( y = 0 \) as well. This special case does not need to be memorized.
Graphing Linear Equations

This method of graphing line equations is not as useful as plotting points. In fact, sometimes it doesn't work at all. An example of this is Example 3 just discussed. It is important to know the definitions of \( x \)- and \( y \)-intercepts. An intercept is where a graph crosses an axis.

This example is not in the book.

The \( x \)-intercept

The graphed line to the left crosses the \( x \)-axis at \(-2\). The coordinates of that point are \( x = -2 \) and \( y = 0 \). The ordered pair is \((-2, 0)\).

The \( y \)-intercept

The graphed line to the left crosses the \( y \)-axis at \(6\). The coordinates of that point are \( x = 0 \) and \( y = 6 \). The ordered pair is \((0, 6)\).

More simply, we can say the \( x \)-intercept is \(-2\) and the \( y \)-intercept is \(6\).

Definitions to remember:

The \( x \)-intercept is where \( y \) equals zero.

The \( y \)-intercept is where \( x \) equals zero.

Graph the equation \(3y = 6x + 12\) by plotting the \( x \)- and \( y \)-intercepts.

Make a table. Again, the table is simply to help keep the numbers organized. It is not a requirement, just a good idea.

\[
\begin{array}{c|c}
\text{x} & \text{y} \\
0 & 0 \\
\end{array}
\]

This time, the table will start out looking like this:

\[
\begin{array}{c|c}
\text{x} & \text{y} \\
\end{array}
\]

The equation will be solved for \( y \) by substituting \( x = 0 \) and also solved for \( x \) by substituting \( y = 0 \). Then the values are put in the table.
$$3y = 6x + 12$$

$$x = 0 \Rightarrow 3y = 6 \cdot 0 + 12$$

$$3y = 12$$

$$\frac{3y}{3} = \frac{12}{3}$$

$$y = 4$$

$$y = 0 \Rightarrow 3y = 6 \cdot x + 12$$

$$0 = 6 \cdot x + 12$$

$$-12 = 6x$$

$$\frac{-12}{6} = x$$

Now the table looks like this:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

After the second ordered pair (-2, 0) is in the table, a third value for x is chosen. If it was chosen earlier, the value might duplicate the x value found when y = 0. This third value for x will be substituted in the line equation and a third point, a “check” point will be found.

At this point, several values could be chosen. The value between 0 and -2, -1 would be OK. Or since we have zero and a negative value, a positive value like +1 would be OK. The choice of +2 would probably be OK as well and might keep us from having to deal with fractions.

Let’s choose +1

$$x = 1$$

Substitute $$x = 1$$ in the line equation.

$$3y = 6 \cdot 1 + 12$$

$$3y = 6 + 12$$

$$3y = 18$$

$$\frac{3y}{3} = \frac{18}{3}$$

$$y = 6$$

The final table:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

These 3 points

$$(0, 4) (-2, 0) (1, 6)$$

will be graphed.
Graph the equation $2x + 5y = 12$ by finding the $x$- and $y$-intercepts. This problem can be done just like in the book. The table would look like this:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.4</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Then like this:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.4</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Notice that $\frac{12}{5}$ is not easy to graph, even when changed to $2.4$. This is one of the drawbacks of this method.

Graph the equation $y = 20x + 60$

This problem can be done just like in the book. The initial table would look like this:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
</tbody>
</table>

The value for $y$ of 60 when $x = 0$ shows that the scale of one or both axes may need to be adjusted to fit the line on the graph. This method can help determine how big the scale must be.

Notice that the check point value for $x$ chosen was 3. This was probably because the value "-3" when $y = 0$ was the result. Choosing -3 was probably done to avoid a fractional answer.
Graph Horizontal and Vertical Lines.

These 2 types of lines are special cases of lines.

p. 416 Ex 7

Graph the equation \( y = 3 \).

Perhaps the directions should read "Graph the line \( y = 3 \)."

This equation could also be written as \( y = 3 + 0x \).

For every value of \( x \), \( y = 3 \).

The error students make with problems like this is to only plot the point \((0, 3)\). The entire horizontal line at \( y = 3 \) is the solution.

\[ \downarrow \quad \downarrow \quad \downarrow \]

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    axis lines=middle,
    grid=both,
    xlabel={\(X\)},
    ylabel={\(Y\)},
    xmin=-5,xmax=5,
    ymin=-5,ymax=5,
    xtick={-5,-4,...,5},
    ytick={-5,-4,...,5},
    xticklabels={-5,-4,...,5},
    yticklabels={-5,-4,...,5},
    enlargelimits=false
]
\addplot[black, thick, ->] coordinates {(-5,3) (5,3)};
\addplot[black, thick, ->] coordinates {(3,-5) (3,5)};
% Add grid lines here
\end{axis}
\end{tikzpicture}
\end{center}

p. 416 Ex 8

Graph the equation \( x = -2 \).

Perhaps the directions should read "Graph the line \( x = -2 \)."

This equation could also be written as \( 0y + x = -2 \).

For every value of \( y \), \( x = -2 \).

The error students make with problems like this is to only plot the point \((-2, 0)\). The entire vertical line at \( x = -2 \) is the solution.

\[ \uparrow \quad \uparrow \quad \uparrow \]
This first section that uses plotting of points is VERY IMPORTANT. This skill translates to drawing or graphing many shapes - lines, circles, curves, line segments and many special curved shapes. The process is not difficult. It involves substitution.

1. Solve the line (linear) equation for \( y \) (so \( y \) is by itself on one side of the equation. For graphing lines, this step is optional, but usually makes the next step easier.

2. Select values for \( x \) to substitute in the line equation and then find the corresponding \( y \) values. The value zero should always be chosen because the work will be easy. Ideally, a positive and a negative value are also chosen. Use order of operations and/or algebra to find the \( y \)-values. This results in ordered pairs that should be recorded in a table to make plotting the points easier.

3. Plot the points using the ordered pairs. They should “line up”. If they don’t appear collinear, a mistake has been made in the algebra or the graphing.

4. Draw the line and put arrows on both ends.

Possible problems - With a pre-mode graph, the points could be off the graph. Choose other values for \( x \).

Fractions in the line equation may give fractions in the points to be graphed. Choosing multiples of the denominator will prevent this and will be shown as it occurs in the examples.

You may wonder, “How do I know what numbers to choose?” Always choose 0 (zero) and be willing to adjust the other 2 choices if the point is off the graph or fractional.

“Two points determine a line” is what is taught in geometry, but we use a third point as a check. Curves plotted in other algebra classes will require 3, 4, 5 or more points to be graphed.