SUBSTITUTION WITH BINOMIALS:

FACTOR \((x-7)^2 + 6(x-7) - 40\) \(\text{LET } u = (x-7)\)

SUBSTITUTE AND FACTOR \(u^2 + 6u - 40\)

\((u + 10)(u - 4)\)

REVERSE THE SUBSTITUTION \((u-7) + 10\) \((x-7) - 4\)

SIMPLIFY \((x+3)(x-11)\)

FACTOR \((3x+1)^2 + 4(3x+1) - 5\) \(\text{LET } u = (3x+1)\)

SUBSTITUTE AND FACTOR \(u^2 + 4u - 5\)

\((u + 5)(u - 1)\)

REVERSE THE SUBSTITUTION \((3x+1) + 5\) \((3x+1) - 1\)

SIMPLIFY \((3x+6)(3x)\)

THIS CAN BE FACTORED, FACTOR OUT THE GCF = 3

\(3(x+2)(3x)\)

MULTIPLY THE MONOMIALS

\(9(x+2)\)

TRY THIS: FACTOR \((2x^2 - 7)^2 - 8(2x^2 - 7) + 12\)

THE FACTORED FORM IS \((2x^2 - 9)(2x^2 - 13)\)
A Substitution with Higher Powers

We have done these in class without formally substituting.

Factor $x^{10} - x^5 - 6 = (x^5 - 3)(x^5 + 2)$

We can also use substitution:

\[
\begin{align*}
\text{Factor} & : x^{10} - x^5 - 6 \\
\text{Let } U &= \text{this variable and exponent} \\
U &= x^5, \text{so } U^2 = (x^5)^2 \\
U^2 &= x^{10}
\end{align*}
\]

Substitute: $x^{10} - x^5 - 6$

\[
\begin{align*}
\text{Factor} & : U^2 - U - 6 \\
& = (U - 3)(U + 2) \\
& = (x^5 - 3)(x^5 + 2)
\end{align*}
\]

Reverse the substitution.

Another example: $2x^4 + 9x^2 - 5$  Let $U = x^2, U^2 = x^4$

Substitute + Factor: $2U^2 + 9U - 5$

\[
\begin{align*}
& = (2U - 1)(U + 5) \\
& = (2x^2 - 1)(x^2 + 5)
\end{align*}
\]

Reverse the substitution.
FACTORIZING 4 TERMS BY 3 AND 1 GROUPING

FACTOR THESE TO GET READY: THEY ARE BOTH A DIFFERENCE OF SQUARES

\[ u^2 - 36 = (u+6)(u-6) \]

\[ (x-2y)^2 - 36 = ((x-2y)+6)((x-2y)-6) \]
\[ = (x-2y+6)(x-2y-6) \]

NOW FACTOR \[ x^2 - 4xy + 4y^2 - 36 \]

You can try to group as we have before, like:

\[ x^2 - 4xy + 4y^2 - 36 \]

but it won't work.

Since the first 3 terms are a perfect square trinomial, we will use 3 and 1 grouping:

\[ \frac{x^2 - 4xy + 4y^2}{(x-2y)^2 - 36} \]

FACTOR:

\[ (x-2y)(x-2y) \]

\[ (x-2y)^2 - 36 \]

THIS IS A DIFFERENCE OF SQUARES

NOW FACTOR LIKE WE DID ABOVE:

\[ (x-2y)^2 - 36 = ((x-2y)+6)((x-2y)-6) \]
\[ = (x-2y+6)(x-2y-6) \]

ANOTHER EXAMPLE:

FACTOR: \[ x^2 + 14x + 49 - y^2 \]

\[ = (x+7)(x+7) \]

\[ (x+7)^2 - y^2 = ((x+7)+y)((x+7)-y) \]
\[ = (x+7+y)(x+7-y) \]
Some 4 term polynomials can be factored using 1 and 3 grouping:

**Factor** \( 100 - x^2 - 10x - 25 \)

Here the last 3 terms are in the form of a perfect square trinomial, but we have to factor out -1.

\[
100 - 1(x^2 + 10x + 25) = 100 - (x+5)^2
\]

Or just

\[
100 - (x+5)^2 \quad \text{This is a difference of squares}
\]

\[
\Rightarrow (10 + (x+5))(10 - (x+5)) \Rightarrow \text{These ( ) are important!}
\]

\[
= (10 + x + 5)(10 - x - 5) = (x + 15)(-x + 5)
\]