Intermediate Algebra Solving Quadratic Equations
Chapter 8

A quadratic equation is in the form:

$$ax^2 + bx + c = 0$$

where a, b and c are real numbers.

We have solved quadratic equations using the zero product rule:

If \((a)(b) = 0\), then either a=0 or b=0.

Example: \(x^2 - 5x - 14 = 0\)

\((x-7)(x+2)=0\)

Solutions: \(x = 7, -2\)

If the quadratic does not factor, we cannot use this method. In Chapter 8 we will learn some other methods for solving quadratic equations.

Section 8.1 Using the Square Root Property
and Completing the Square to Solve Quadratic Equations

\[ \sqrt{\text{Solve equations using the square root property.}} \]

\[ \text{If } \alpha \text{ and } \beta \text{ are complex numbers} \]

\[ \text{and } \alpha^2 = \beta, \text{ then } \alpha = \pm \sqrt{\beta} \]

Examples:

Solve \(m^2 = 64\)

\[
\begin{align*}
m &= \pm \sqrt{64} \\
m &= \pm 8
\end{align*}
\]

Check:

\[
\begin{align*}
(8)^2 &= 64 \\
64 &= 64 \checkmark \\
(-8)^2 &= 64 \\
64 &= 64 \checkmark
\end{align*}
\]

Solve \(x^2 = 50\)

\[
\begin{align*}
x &= \pm \sqrt{50} \\
x &= \pm 5\sqrt{2}
\end{align*}
\]

Check:

\[
\begin{align*}
(5\sqrt{2})^2 &= 50 \\
25(2) &= 50 \\
50 &= 50 \checkmark \\
(-5\sqrt{2})^2 &= 50 \\
25(2) &= 50 \\
50 &= 50 \checkmark
\end{align*}
\]
SOLVE \( 3x^2 - 54 = 0 \)

First isolate the squared expression:

\[
\begin{align*}
3x^2 &= 54 \\
\frac{3x^2}{3} &= \frac{54}{3} \\
x^2 &= 18
\end{align*}
\]

Now apply the sq. rt. prop.

\[
\begin{align*}
x &= \pm \sqrt{18} \\
x &= \pm 3\sqrt{2}
\end{align*}
\]

Here is an example with a more complicated squared expression:

SOLVE \( (3k+1)^2 = 2 \) Apply the sq. rt. property

\[
\begin{align*}
3k + 1 &= \pm \sqrt{2} \\
solve \ for \ k
\end{align*}
\]

\[
3k = -1 \pm \sqrt{2}
\]

\[
\begin{align*}
\frac{3k}{3} &= \frac{-1 \pm \sqrt{2}}{3} \\
k &= -\frac{1}{3} \pm \frac{\sqrt{2}}{3}
\end{align*}
\]

Either form is acceptable.

SOLVE:

\((x+7)^2 = 12\)

\[
\begin{align*}
x+7 &= \pm \sqrt{12} \\
x+7 &= \pm 2\sqrt{3} \\
x &= -7 \pm 2\sqrt{3}
\end{align*}
\]
SOLVE EQUATIONS BY COMPLETING THE SQUARE

IN THE LAST TWO EXAMPLES THERE WAS A SQUARED BINOMIAL LIKE \((3k+1)^2\) AND \((m+7)^2\)

SO WE WILL PRACTICE FORMING AND FACTORING PERFECT SQUARE TRINOMIALS TO GET SQUARED BINOMIALS:

This activity is preparation for solving quadratic equations by

CLASSROOM ACTIVITY 8.1B

Completing the Square

Add the appropriate constant to form a perfect square trinomial. Then factor the result.

1. \(t^2 + 20t + 100 = \) \(\text{factor} \rightarrow \)
   \((t+10)(t+10) = (t+10)^2\

2. \(p^2 - 10p + 25 = \) \(\text{factor} \rightarrow \)
   \((p-5)(p-5) = (p-5)^2\)

3. \(m^2 + 7m + \frac{49}{4} = \) \(\text{factor} \rightarrow \)
   \((m + \frac{7}{2})(m + \frac{7}{2}) = (m + \frac{7}{2})^2\)

4. \(a^2 + \frac{2}{9}a + \frac{1}{81} = \) \(\text{factor} \rightarrow \)
   \((a + \frac{1}{9})(a + \frac{1}{9}) = (a + \frac{1}{9})^2\)
IF THE EQUATION CONTAINS A PERFECT SQUARE TRINOMIAL
WE CAN SOLVE USING THE SQUARE ROOT PROPERTY:

SOLVE:
\[ x^2 - 10x + 25 = 11 \]
\[ (x-5)(x-5) = 11 \]
\[ (x-5)^2 = 11 \]
\[ x-5 = \pm \sqrt{11} \]
\[ x = 5 \pm \sqrt{11} \]

EXAMPLE:
\[ x^2 + 14x - 1 = 0 \]

CANNOT BE SOLVED BY FACTORING

\[ \begin{array}{c}
\hline
x^2 + 14x = 1 & \text{ADD 1 TO EACH SIDE TO GET READY TO COMPLETE THE SQUARE} \\
\hline
x^2 + 14x + 49 = 1 + 49 & \text{ADD } (14/2)^2 = (7)^2 = 49 \text{ TO EACH SIDE TO COMPLETE THE SQUARE} \\
(x+7)(x+7) = 50 & \text{FACTOR LEFT SIDE, SIMPLIFY RIGHT} \\
(x+7)^2 = 50 & \\
\hline
x+7 = \pm \sqrt{50} & \text{APPLY SQ. RT. PROPERTY} \\
x+7 = \pm 5\sqrt{2} & \\
\hline
x = -7 \pm 5\sqrt{2} & \text{SOLVE FOR } x
\end{array} \]
**Example:** If the leading coefficient is not 1:

<table>
<thead>
<tr>
<th>Mathematical Steps</th>
<th>Explain the Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x^2 + 40x + 5 = 0)</td>
<td>Get in standard form.</td>
</tr>
<tr>
<td>(\frac{4x^2 + 40x + 5}{4} = 0)</td>
<td>Divide by 4 to get leading coefficient = 1</td>
</tr>
<tr>
<td>(x^2 + 10x + \frac{5}{4} = 0)</td>
<td></td>
</tr>
<tr>
<td>(x^2 + 10x = -\frac{5}{4})</td>
<td>Move constant to right to get ready to complete the square.</td>
</tr>
<tr>
<td>(x^2 + 10x + 25 = -\frac{5}{4} + \frac{25}{4})</td>
<td>Add (10 \cdot \frac{1}{2}) to both sides.</td>
</tr>
<tr>
<td>((x+5)^2 = \frac{95}{4})</td>
<td>Factor left side and simplify right side.</td>
</tr>
<tr>
<td>(x+5 = \pm \sqrt{\frac{95}{4}})</td>
<td>Apply sq. root property</td>
</tr>
<tr>
<td>(x = -5 \pm \frac{\sqrt{95}}{2})</td>
<td>Simplify and solve for x</td>
</tr>
</tbody>
</table>