Example of a System of Linear Equations

At the local High school football game you go to the concession stand with your friend Mel to buy hotdogs and sodas. You buy 3 hotdogs and 4 sodas, and pay a total of $24. Mel buys 1 hotdog and 2 sodas and pays a total of $10.

We can use this information to figure out the price of a hotdog and the price of a soda.

Let \( x \) = the price of a hotdog

Let \( y \) = the price of a soda

Equation representing your purchase:

\[
3x + 4y = 24
\]

Equation representing Mel’s purchase:

\[
x + 2y = 10
\]

There are many possible prices that would satisfy your purchase, and many possible prices that would satisfy Mel’s purchase, but only one combination that satisfies both. This combination is the solution of the system of two equations.

\[
\begin{align*}
3x + 4y &= 24 \\
x + 2y &= 10
\end{align*}
\]

Graph each line and find the intersection.
NOTES - SECTION 3.1  Systems of Linear Equations

Decide whether an ordered pair is a solution of a linear system.

Is (3, -12) a solution of the system:

<table>
<thead>
<tr>
<th>2x + y = -6</th>
<th>(3, -12) is NOT a solution of the system.</th>
</tr>
</thead>
<tbody>
<tr>
<td>x + 3y = 2</td>
<td>c_2</td>
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</table>

Does \( 2(3) + (-12) = -6 \)? YES

Does \( 3 + 3(-12) = 2 \)? NO

Is (-4, 2)?

Does \( 2(-4) + 2 = -6 \)? YES

Does \(-4 + 3(2) = 2 \)? YES

(-4, 2) is a solution of the system.

Solve linear systems by graphing.

Solve this system by graphing.

\[
\begin{align*}
2x + y &= -5 \\
-x + 3y &= 6
\end{align*}
\]

1) Graph: \(2x + y = -5\)

\[
\begin{align*}
y &= -2x - 5 \\
m &= -2, \\
y \text{ int} &= (0, -5)
\end{align*}
\]

2) Graph \(-x + 3y = 6\)

The point where the lines intersect is the solution.

- If \(x = 0\), \(y = 2\) \((0, 2)\)
- If \(y = 0\), \(x = -6\) \((-6, 0)\)
- If \(x = 3\), \(y = 3\) \((3, 3)\)

The point where the lines intersect is the solution.

A system is CONSISTENT if: it has at least one solution

A system is INCONSISTENT if: it has no solution (parallel lines)

The equations of a system are DEPENDENT if: any solution of one is also a solution of the other (same lines)

The equations of a system are INDEPENDENT if: they share at most one solution (different lines)