SOLVING SYSTEMS OF EQUATIONS WITH 3 VARIABLES

SYSTEM:
\[ A: \quad -3x + y - z = -10 \]
\[ B: \quad -4x + 2y + 3z = -1 \]
\[ C: \quad 2x + 3y - 2z = -5 \]

Decide which variable to eliminate first. Either \( y \) or \( z \) would be good choices here, since in \[ A \] their coefficients are 1 and -1.

Let's eliminate \( z \). We will have to do this twice.

Using \[ A \] and \[ B: \]
\[ 3A \]
\[ -9x + 3y - 3z = -30 \]
\[ B \]
\[ -4x + 2y + 3z = -1 \]

Let's call this \( D \)
\[ -13x + 5y = -31 \]

Using \[ A \] and \[ C: \]
\[ -2A \]
\[ 6x - 2y + 2z = 20 \]
\[ C \]
\[ 2x + 3y - 2z = -5 \]

Let's call this \( E \)
\[ 8x + y = 15 \]

Now use \( D \) and \( E \) to eliminate another variable.

\[ D \]
\[ -13x + 5y = -31 \]
\[ -5E \]
\[ 40x - 5y = -75 \]
\[ -53x = -106 \]
\[ x = 2 \]

Plug \( x = 2 \) into \[ A \] before to find \( z \).

\[ A \]
\[ -3(2) + 2 + z = -10 \]
\[ -7 + z = -10 \]
\[ z = 3 \]

Plug \( x = 2 \) into \( E \) or \( F \) to find \( y \).

\[ E \]
\[ 8(2) + y = 15 \]
\[ 16 + y = 15 \]
\[ y = -1 \]

Solution: \( (2, -1, 3) \)