Section 5.6
Solving Quadratic Equations using Factoring

Standard form of a quadratic Equation: \( ax^2 + bx + c = 0 \)

Zero Factor Property: If \((a)(b) = 0\), then either \(a=0\) or \(b=0\).

To solve a quadratic equation by factoring:

1) Write the equation in standard form.
2) Factor the left side.
3) Set each factor equal to zero and solve each linear equation.

Examples:

1) \( x^2 - 5x - 14 = 0 \)
   \((x - 7)(x + 2) = 0\)
   \(x - 7 = 0\)
   \(x + 2 = 0\)
   \(x = 7\)
   \(x = -2\)

2) \(-2x - 15 = -x^2\)
   \(x^2 - 2x - 15 = 0\)
   \((x - 5)(x + 3) = 0\)
   \(x - 5 = 0\)
   \(x + 3 = 0\)
   \(x = 5\)
   \(x = -3\)

3) \(15x^2 - 10x = 0\)
   \(5x(3x - 2) = 0\)
   \(5x = 0\)
   \(3x - 2 = 0\)
   \(x = 0\)
   \(x = \frac{2}{3}\)

4) \(3x^2 - 4 = -4x\)
   \(3x^2 + 4x - 4 = 0\)
   \((3x - 2)(x + 2) = 0\)
   \(3x - 2 = 0\)
   \(x + 2 = 0\)
   \(x = \frac{2}{3}\)
   \(x = -2\)

5) \(x^2 = 64\)
   \(-64 = -64\)
   \((x + 8)(x - 8) = 0\)
   \(x = 8\)
   \(x = -8\)

6) \((x + 2)(x + 5) = 0\)
   \(x + 2 = 0\)
   \(x + 5 = 0\)
   \(x = -2\)
   \(x = -5\)

7) \((x + 2)(x + 5) = -2\)
   \(\text{MUST GET IN STANDARD FORM}\)
   \(x^2 + 5x + 2x + 10 = -2\)
   \(x^2 + 7x + 10 = -2\)
   \(x^2 + 7x + 12 = 0\)
   \(\text{LHS} \rightarrow \text{RHS}\)
   \(x + 3 = 0\)
   \(x + 4 = 0\)
   \(x = -3\)
   \(x = -4\)
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Solving Problems Using Factoring

Write expressions for:

a) 3 times a number. \( 3x \)

b) 7 more than a number. \( x + 7 \)

c) Length is 5 less than width. \( \text{LENGTH} = w - 5 \)

d) Length is 4 less than twice the width. \( \text{LENGTH} = 2w - 4 \)

Number Problems

The product of two positive integers is 64. One integer is 4 times the other. Find both integers.

The product of two consecutive positive integers is 56. Find the two integers.

Area of a rectangle

The length of a rectangle is 3 less than 3 times the width. The area is 90. Find the length.
The length of a rectangle is 4 more than twice the width. Find the width of the rectangle if the area is 6.

\[ 2w + 4 \]

**LENGTH \times WIDTH = AREA**

\[
(2w + 4)(w) = 6 \\
2w^2 + 4w = 6 \\
2w^2 + 4w - 6 = 0 \\
2w^2 + 4w - 6 = 0
\]

\[ \Rightarrow \frac{2w + 3}{w - 1} = 0 \]

\[ \Rightarrow 2w + 3 = 0 \]

\[ w = -\frac{3}{2} \]

\[ w = 1 \]

**USE THE POSITIVE VALUE SINCE W IS THE SIDE OF A RECTANGLE.**

\[ w = 1 \]

\[ l = 2(1) + 4 = 6 \]

**Pythagorean Theorem**

If \( a \) and \( b \) are the legs of a right triangle, and \( c \) is the hypotenuse, then \( a^2 + b^2 = c^2 \). **Hypotenuse is the side across from the right angle**

Can 2, 3, 4 be the sides of a right triangle?

\[ 2^2 + 3^2 = 4^2 \? \text{ NO} \]

Can 3, 4, 5?

\[ 3^2 + 4^2 = 5^2 \? \text{ YES} \]

Use the Pythagorean theorem to find the value of \( x \) for each triangle.

**Diagram 1**

\[ x \]

\[ 5 \]

\[ 12 \]

\[ 5^2 + 12^2 = x^2 \]

\[ 144 = 121 \]

\[ x^2 = 169 \]

\[ x = \sqrt{169} \]

\[ x = 13 \]

**Diagram 2**

\[ x \]

\[ 2x + 6 \]

\[ x = \sqrt{(2x + 4)^2} \]

\[ x^2 + 4x^2 + 16x + 16 = (2x + 6)^2 \]

\[ 5x^2 + 16x + 16 = 4x^2 + 24x + 36 \]

\[ 5x^2 + 16x + 16 = 4x^2 + 24x + 36 \]

\[ -x^2 - 8x - 20 = 0 \]

\[ x = 10 \]

\[ x = 2 \]

**Diagram 3**

\[ x \]

\[ x \]

\[ x \]

\[ x \]

**SIDE CANNOT BE NEGATIVE.**