The multiplication rules can be used to find the probability of two or more events that occur in sequence.

\[
\begin{array}{c}
HH \\
HT \\
TH \\
TT
\end{array}
\]

\[
P(\text{TT}) = \frac{1}{4}
\]

**Independent Events**

Two events A and B are independent events if the fact that A occurs does not affect the probability of B occurring.

When two events are independent, the probability of both occurring is

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]

\[
P(T \text{ and } T) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}
\]

**Dependent Events**

Two events are said to be dependent events if the occurrence of the first event affects the occurrence of the second event in such a way that the probability is changed.

The conditional probability of an event B in a relationship to an event A is the probability that event B occurs after event A has already occurred.

\[
P(B|A) \quad \text{read as “the probability of B given A has occurred”}
\]

When two events are dependent, the probability of both occurring is

\[
P(A \text{ and } B) = P(A) \cdot P(B|A)
\]

\[
P(\text{Ace and then Ace}) = \frac{4}{52} \cdot \frac{3}{52} = \frac{1}{221}
\]

The key word AND tells us to MULTIPLY probabilities.

\[
P(\text{Good then Good}) = \frac{5}{8} \cdot \frac{4}{7}
\]

For large populations, assume replacement.
Conditional Probability

The probability that the second event $B$ occurs given that the first event $A$ has occurred is

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

- $P(\text{online}) = .38$
- $P(\text{online and on campus}) = .30$
- $P(\text{on campus|online}) = \frac{.30}{.38}$

Probabilities for “At Least”

The complement of “at least one” is “none”, so

$$P(\text{at least one}) = 1 - P(\text{none})$$

- $P(\text{online}) = .38$
- $P(\text{online}) = .62$
- $P(3 \text{ online}) = .38^3$
- $P(3 \text{ not online}) = .62^3$

- $P(\text{at least one girl}) = 1 - P(\text{no girls})$
  - $= 1 - \frac{1}{8}$
  - $= \frac{7}{8}$

Select 3:

$$P(\text{at least 1 online}) = 1 - P(\text{none online})$$

$$= 1 - .62^2$$
Section 4-4 Examples

Exercise #4  INDEPENDENT EVENTS
The Gallup Poll reported that 52% of Americans used a seat belt the last time they got into a car. If four people are selected at random, find the probability that all used a seat belt the last time they got into a car.

\[
(0.52)^4 = 0.1179 \approx 7.3\% 
\]

Additional questions:
Find the probability that a person did not use a seat belt.

\[
P\left(\text{not used seat belt}\right) = 0.48
\]

If four people are selected at random, find the probability that none of them used a seat belt.

\[
(0.48)^4 = 0.053 \approx 5.3\%
\]

If four people are selected at random, find the probability that at least one of them used a seat belt.

\[
P(\text{at least 1 of 4 used seat belt}) = 1 - P(\text{none of 4 used seat belt})
= 1 - (0.48)^4
= 0.947 \approx 94.7\%
\]

Exercise #8  DEPENDENT EVENTS
If 2 cards are selected from a standard deck of 52 cards without replacement, find these probabilities.

a) Both are spades.

\[
\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}
\]

b) Both are the same suit.

\[
\frac{4}{4} \cdot \frac{12}{51} = \frac{4}{17}
\]

OR

\[
P(\text{same suit happens}) = \frac{1}{17}
\]

\[
= P(\text{one spade}) + \frac{1}{17}
\]

\[
= P(\text{one heart}) + \frac{1}{17}
\]

\[
= P(\text{one club}) + \frac{1}{17}
\]

\[
= P(\text{one diamond}) + \frac{1}{17}
\]

\[
= \frac{4}{17}
\]

c) Both are kings.

\[
\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}
\]

\[
\frac{14}{52} \cdot \frac{3}{51} = \frac{1}{221}
\]

\[
\frac{13}{52} \cdot \frac{2}{51} = \frac{1}{221}
\]
Exercise #32  CONDIONAL PROBABILITY
In a pizza restaurant, 95% of the customers order pizza. If 65% of the customers order pizza and a salad, find the probability that a customer who orders pizza will also order a salad.

\[
P(S | P) = \frac{P(S \text{ and } P)}{P(P)} = \frac{65}{95} = \frac{13}{19} = .68
\]

Exercise #40  PROBABILITY AT LEAST ONE
A lot of portable radios contains 15 good radios and 3 defective ones. If 2 are selected and tested, find the probability that at least one will be defective.

\[
P(\text{at least one of 2 def}) = 1 - P(\text{none of 2 def})
\]
\[
= 1 - P(S) = 1 - \frac{15 \cdot 14}{18 \cdot 17}
\]
\[
= 1 - \frac{5 \cdot 14}{6 \cdot 17}
\]
\[
= 1 - \frac{35}{51}
\]
\[
= \frac{16}{51}
\]