Section 5-4  Binomial Probability Distributions

A binomial distribution is a probability distribution that has only two outcomes, called a success and a failure. Flipping a coin can only have two outcomes, a head or a tail. Rolling a die has six possible outcomes, but can be thought of as a binomial if you reduce it to only two outcomes, getting the number you want or getting any other number. There are four requirements for a binomial experiment.

Binomial experiments have:

1. a fixed number of trials
2. only two outcomes
3. independent trials
   (the outcome of one does not affect the outcome of the next)
4. a constant probability of success

Which of these involves binomial probability? Identify n, p, q, and list the possible values of the variable for any binomial probabilities.

A certain procedure has an 85% chance of success. A doctor performs the procedure on eight patients. The variable represents the number of successful procedures.

\[
\begin{array}{c|c}
\text{BINOMIAL} & \# \text{ OF TRIALS } M = 8 \\
\hline
0.85 & \text{POSSIBLE VALUES OF VARIABLE} \\
0.15 & 0, 1, 2, 3, 4, 5, 6, 7, 8
\end{array}
\]

A jar contains 5 red marbles and 6 green marbles. You randomly select 3 marbles from the jar without replacement. The variable represents the number of red marbles.

\[
\begin{array}{c|c}
\text{ NOT BINOMIAL, WITHOUT REPLACEMENT MEANS} \\
\text{THE TRIALS ARE NOT INDEPENDENT.}
\end{array}
\]

You take a multiple choice quiz that consists of 5 questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. The variable represents the number of correct answers.

\[
\begin{array}{c|c}
\text{BINOMIAL} & \# \text{OF TRIALS } M = 5 \\
\hline
0.25 & \text{POSSIBLE VALUES OF VARIABLE} \\
0.75 & 0, 1, 2, 3, 4, 5
\end{array}
\]

A political polling organization calls 1,102 people and asks, “Do you approve, disapprove, or have no opinion of the way the mayor is handling his job?” The variable represents the possible answers.

\[
\begin{array}{c|c}
\text{ NOT BINOMIAL, MORE THAN 2 OUTCOMES}
\end{array}
\]

The outcomes of a binomial experiment with the corresponding probabilities is called a binomial distribution.
Binomial Probability

\[ P(X) = \binom{n}{x} \frac{n!}{(n-x)!x!} p^x q^{n-x} \]

When \( n \) = total number of trials
\( p \) = probability of success
\( x \) = number of desired successes
\( q \) = probability of failure

**Binomial Probability Examples**

**Example 1**
For a certain company, there is a 17% chance that an employee will be absent on any given day. If one department has 12 employees, find the probability of

\[ m = 12 \quad p = .17 \quad q = .83 \]

a) no absences on a given day.

\[ P(0) = \binom{12}{0} (.17)^0 (.83)^{12} = 1 (1)(.83)^{12} = .107 \]

b) 3 absences on a given day.

\[ P(3) = \binom{12}{3} (.17)^3 (.83)^9 = 220 (.17)^3 (.83)^9 = .202 \]

**Example 2**
27% of US cars are some shade of blue and there are 10 cars in a parking lot.

\[ m = 10 \quad p = .27 \quad q = .73 \]

a) Find the probability that 5 of the cars will be blue.

\[ P(5) = \binom{10}{5} (.27)^5 (.73)^5 = 252 (.27)^5 (.73)^5 = .075 \]

b) Find the probability that at most 2 are blue.

**AT MOST 2 MEANS 0, 1, 2, OR 2**

\[ P(0) = \binom{10}{0} (.27)^0 (.73)^{10} = 1 (1)(.73)^{10} = .043 \]

\[ P(1) = \binom{10}{1} (.27)^1 (.73)^9 = 10 (.27)(.73)^9 = .159 \]

\[ P(2) = \binom{10}{2} (.27)^2 (.73)^8 = 45 (.27)^2 (.73)^8 = .265 \]

\[ P(\text{AT MOST 2}) = .467 \]
Binomial Probability Examples Using Table B

Example 1
Of a certain community, 40% is Hispanic. If 12 people are selected at random from the community, find the probability that less than 4 are Hispanic.

\[ m = 12, \ p = .40, \ q = .60 \] 
\[ X \ \text{is less than 4} \Rightarrow 0, 1, 2, 3 \]

From Table B
\[ P(0) = .002 \]
\[ P(1) = .017 \]
\[ P(2) = .064 \]
\[ P(3) = .142 \]
\[ P(\text{less than 4}) = .225 \]

Example 2
Of cameras sold in the United States, 10% are manufactured in Europe. If 20 cameras are selected at random, what is the probability that at least 3 were manufactured in Europe?

\[ m = 20, \ p = .10, \ q = .90 \] 
\[ X \ \text{is at least 3} \Rightarrow 3, 4, 5, \ldots, 10 \]

From Table B
\[ P(0) = .122 \]
\[ P(1) = .270 \]
\[ P(2) = .285 \]
\[ \frac{.677}{.677} \]
\[ P(\text{at least 3}) = 1 - .677 = .323 \]
\[ P(\text{at least 3}) = .323 \]

Mean and Standard Deviation for Binomial Probabilities

- Mean : \[ \mu = np \]
- Variance : \[ \sigma^2 = npq \]
- Standard deviation : \[ \sigma = \sqrt{npq} \]

Example 1
If a company has 200 employees and 17% are absent each day, find the mean and standard deviation for the number of absences each day.

\[ m = 200, \ p = .17, \ q = .83 \]

\[ \mu = 200(.17) = 34 \ \text{ABSEES} \]
\[ \sigma = \sqrt{200(.17)(.83)} = 5.3 \ \text{ABSEES} \]

Example 2
Of all people, 56% wear seat belts. If 50 people were randomly checked, find the mean and standard deviation for how many will be wearing their seat belts.

\[ m = 50, \ p = .56, \ q = .44 \]

\[ \mu = 50(.56) = 28 \ \text{PEOPLE} \]
\[ \sigma = \sqrt{50(.56)(.44)} = 3.5 \ \text{PEOPLE} \]