Two functions \( f \) and \( g \) are called inverse functions if

\[
(f \circ g)(x) = (g \circ f)(x) = x.
\]
Example 1.

Verify that $f$ and $g$ are inverse functions. Graph both $f$ and $g$.

$$f(x) = 3x + 4 \quad g(x) = \frac{x - 4}{3}$$
Example 1.

Verify that $f$ and $g$ are inverse functions. Graph both $f$ and $g$.

\[ f(x) = 3x + 4 \quad g(x) = \frac{x - 4}{3} \]

\[ (f \circ g)(x) = f(g(x)) \]

\[ = f \left( \frac{x - 4}{3} \right) \]

\[ = 3 \left( \frac{x - 4}{3} \right) + 4 \]

\[ = (x - 4) + 4 \]

\[ = x \]
Verify that \( f \) and \( g \) are inverse functions. Graph both \( f \) and \( g \).

\[
f(x) = 3x + 4 \quad g(x) = \frac{x - 4}{3}
\]

\[
(g \circ f)(x) = g(f(x))
\]

\[
= g(3x + 4)
\]

\[
= \frac{(3x + 4) - 4}{3}
\]

\[
= \frac{3x}{3}
\]

\[
= x
\]
Example 2.

Verify that $f$ and $g$ are inverse functions. Graph both $f$ and $g$.

$$f(x) = x^3 + 2 \quad g(x) = \sqrt[3]{x} - 2$$
Recall the definition of a function:
Recall the definition of a function: A *function* is a specific type of a relation where each element in the domain corresponds to exactly one element in the range.
A function $f$ is called *one-to-one* if each element in the range corresponds to exactly one element in the domain. If a function is one-to-one, then it has an inverse function.
Example 3.

Which of the following functions are one-to-one?

One-to-one:
Example 3.

Which of the following functions are one-to-one?

One-to-one: No
Example 3. (Continued)

Which of the following functions are one-to-one?

One-to-one:
Which of the following functions are one-to-one?

One-to-one: Yes
Which of the following functions are one-to-one?

One-to-one:
Which of the following functions are one-to-one?

One-to-one: Yes
Which of the following functions are one-to-one?

One-to-one:
Which of the following functions are one-to-one?

One-to-one: Yes
Example 3. (Continued)

Which of the following functions are one-to-one?

One-to-one:
Which of the following functions are one-to-one?

One-to-one: No
Example 4.

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

$$f(x) = \frac{1}{2}x - 3$$
Example 4.

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

$$f(x) = \frac{1}{2}x - 3$$

$$y = \frac{1}{2}x - 3$$

$$x = \frac{1}{2}y - 3$$

$$x + 3 = \frac{1}{2}y$$

$$2x + 6 = y$$

$$f^{-1}(x) = 2x + 6$$
Example 4. (Continued)

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

$$f(x) = \frac{1}{2}x - 3$$

$$f^{-1}(x) = 2x + 6$$

Domain of $f = (-\infty, \infty)$

Range of $f = (-\infty, \infty)$

Domain of $f^{-1} = (-\infty, \infty)$

Range of $f^{-1} = (-\infty, \infty)$
Example 5.

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

$$f(x) = x^2 - 3, \quad x \geq 0$$
Example 5.

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

$$f(x) = x^2 - 3, \quad x \geq 0$$

$$y = x^2 - 3$$

$$x = y^2 - 3$$

$$x + 3 = y^2$$

$$\sqrt{x + 3} = y$$

$$f^{-1}(x) = \sqrt{x + 3}$$
Example 5. (Continued)

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

\[ f(x) = x^2 - 3, \quad x \geq 0 \]

\[ f^{-1}(x) = \sqrt{x + 3} \]

Domain of $f = [0, \infty)$

Range of $f = [-3, \infty)$

Domain of $f^{-1} = [-3, \infty)$

Range of $f^{-1} = [0, \infty)$
Example 6.

Determine the inverse of the one-to-one function. State the domain and range of \( f \) and \( f^{-1} \).

\[
f(x) = \sqrt{x - 4}
\]
Example 6.

Determine the inverse of the one-to-one function. State the domain and range of \( f \) and \( f^{-1} \).

\[ f(x) = \sqrt{x - 4} \]

\[ y = \sqrt{x - 4} \]

\[ x = \sqrt{y - 4} \]

\[ x^2 = y - 4 \]

\[ x^2 + 4 = y \]

\[ f^{-1}(x) = x^2 + 4 \]
Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

\[ f(x) = \sqrt{x - 4} \]

\[ f^{-1}(x) = x^2 + 4 \]

Domain of $f = [4, \infty)$

Range of $f = [0, \infty)$

Domain of $f^{-1} = [0, \infty)$

Range of $f^{-1} = [4, \infty)$
Example 7.

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

\[ f(x) = \sqrt[3]{x} + 2 \]
Example 7.

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

$$f(x) = \sqrt[3]{x} + 2$$

$$y = \sqrt[3]{x} + 2$$

$$x = \sqrt[3]{y} + 2$$

$$x^3 = y + 2$$

$$x^3 - 2 = y$$

$$f^{-1}(x) = x^3 - 2$$
Example 7. (Continued)

Determine the inverse of the one-to-one function. State the domain and range of $f$ and $f^{-1}$.

\[ f(x) = \sqrt[3]{x} + 2 \]

\[ f^{-1}(x) = x^3 - 2 \]

Domain of $f = (-\infty, \infty)$

Range of $f = (-\infty, \infty)$

Domain of $f^{-1} = (-\infty, \infty)$

Range of $f^{-1} = (-\infty, \infty)$