Quadratic Equations

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A quadratic equation is an equation that can be written as

$$ax^2 + bx + c = 0$$

where $a$, $b$, and $c$ are real numbers and $a \neq 0$. 
If $a \cdot b = 0$, then $a = 0$ or $b = 0$. 
Example 1.

Solve.

\[ x^2 - 5x + 6 = 0 \]

Thus, by the zero factor property, either

\[ x - 2 = 0 \] or \[ x - 3 = 0 \]

\[ x = 2 \] \[ x = 3 \]

The solution set is \{2, 3\}.
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\[ 3x(x - 2) = 4(x + 1) + 4 \]
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Solution.

\[ 3x(x - 2) = 4(x + 1) + 4 \]

\[ 3x^2 - 6x = 4x + 4 + 4 \]

\[ 3x^2 - 10x - 8 = 0 \]

\[ (3x + 2)(x - 4) = 0 \]

\[ 3x + 2 = 0 \quad \text{or} \quad x - 4 = 0 \]

\[ x = -\frac{2}{3} \quad \text{or} \quad x = 4 \]

The solution set is \( \left\{ -\frac{2}{3}, 4 \right\} \).
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\[ (3x + 2)(x - 4) = 0 \]

\[ 3x + 2 = 0 \quad x - 4 = 0 \]

\[ x = -\frac{2}{3} \quad x = 4 \]

The solution set is \( \left\{ -\frac{2}{3}, 4 \right\} \).
If $x^2 = a$, then $x = \pm \sqrt{a}$. 
Example 3.

Solve.

\[ 3x^2 + 4 = 58 \]

Solution.

\[ 3x^2 = 54 \]
\[ x^2 = 18 \]
\[ x = \pm \sqrt{18} \]
\[ x = \pm 3\sqrt{2} \]

The solution set is \( \{ -3\sqrt{2}, 3\sqrt{2} \} \).
Example 3.

Solve.

\[3x^2 + 4 = 58\]

Solution.

\[3x^2 + 4 = 58\]

The solution set is \(\{\text{ }3\sqrt{2}, -3\sqrt{2}\}\).
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The solution set is \(\{-3\sqrt{2}, 3\sqrt{2}\}\).
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\[ 3x^2 = 54 \]

\[ x^2 = 18 \]

\[ \sqrt{x^2} = \pm \sqrt{18} \]

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$3x^2 + 4 = 58$

Solution.

$3x^2 + 4 = 58$

$3x^2 = 54$

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$x = \pm 3\sqrt{2}$

The solution set is $\{-3\sqrt{2}, 3\sqrt{2}\}$.
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\[(x - 3)^2 = 4\]

The solution set is \{1, 5\}. 

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\[\sqrt{(x - 3)^2} = \pm \sqrt{4}\]

\[x - 3 = \pm 2\]

The solution set is \(\{1, 5\}\).
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\[(x - 3)^2 = 4\]

\[\sqrt{(x - 3)^2} = \pm \sqrt{4}\]

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\[x = 3 \pm 2\]
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The solution set is \(\{1, 5\}\).
Example 5.

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\[\sqrt{(2x - 1)^2} = \pm \sqrt{-5}\]

The solution set is

\[\{1 + i\sqrt{5}, 1 - i\sqrt{5}\}\].
Example 5.

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\[(2x - 1)^2 = -5\]

Solution.

\[(2x - 1)^2 = -5\]

\[\sqrt{(2x - 1)^2} = \pm \sqrt{-5}\]

\[2x - 1 = \pm i\sqrt{5}\]

The solution set is

\[\{1 + i\sqrt{5}, 1 - i\sqrt{5}\}\]
Example 5.

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\[(2x - 1)^2 = -5\]

Solution.

\[(2x - 1)^2 = -5\]

\[\sqrt{(2x - 1)^2} = \pm \sqrt{-5}\]

\[2x - 1 = \pm i \sqrt{5}\]

\[2x = 1 \pm i \sqrt{5}\]

The solution set is \(\{1 + i \sqrt{5}/2, 1 - i \sqrt{5}/2\}\).
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\[(2x - 1)^2 = -5\]

\[\sqrt{(2x - 1)^2} = \pm \sqrt{-5}\]

\[2x - 1 = \pm i\sqrt{5}\]

\[2x = 1 \pm i\sqrt{5}\]

\[x = \frac{1 \pm i\sqrt{5}}{2}\]
Example 5.

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\[(2x - 1)^2 = -5\]

\[\sqrt{(2x - 1)^2} = \pm \sqrt{-5}\]

\[2x - 1 = \pm i\sqrt{5}\]

\[2x = 1 \pm i\sqrt{5}\]

\[x = \frac{1 \pm i\sqrt{5}}{2}\]

The solution set is \[\left\{ \frac{1 + i\sqrt{5}}{2}, \frac{1 - i\sqrt{5}}{2} \right\}\].
For any quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
Example 6.

Solve.

$$3x^2 - 5x - 2 = 0$$
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Solve.

\[3x^2 - 5x - 2 = 0\]

Solution.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

The solution set is \[\{-\frac{1}{3}, 2\}\].
Example 6.

Solve.

\[3x^2 - 5x - 2 = 0\]

**Solution.**

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[= \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)}\]

The solution set is \([-1, 2]\).
Example 6.

Solve.

\[ 3x^2 - 5x - 2 = 0 \]

Solution.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
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\[
x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)}
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\[
x = \frac{5 \pm \sqrt{25 + 24}}{6}
\]

The solution set is \{\(-1/3, 2\)\}. 
Example 6.

Solve.

$$3x^2 - 5x - 2 = 0$$

Solution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25 + 24}}{6}$$

$$= \frac{5 \pm \sqrt{49}}{6}$$

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$$= \frac{5 \pm \sqrt{49}}{6} = \frac{5 \pm 7}{6}$$

The solution set is $$\{-1, 2\}$$.

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\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

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\[= \frac{5 \pm \sqrt{25 + 24}}{6}\]

\[= \frac{5 \pm \sqrt{49}}{6} = \frac{5 \pm 7}{6}\]

The solution set is \(\{-\frac{1}{3}, 2\}\).
Example 7.

Solve.

\[ x^2 - 3x - 7 = 0 \]

Solution.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ a = 1, \quad b = -3, \quad c = -7 \]

\[ x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-7)}}{2(1)} \]

\[ x = \frac{3 \pm \sqrt{9 + 28}}{2} \]

\[ x = \frac{3 \pm \sqrt{37}}{2} \]

The solution set is \( \{ \frac{3 + \sqrt{37}}{2}, \frac{3 - \sqrt{37}}{2} \} \).
Example 7.

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\[ x^2 - 3x - 7 = 0 \]

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