5.4 Dividing Polynomials

Objectives

1. Divide a polynomial with multiple terms by a monomial.
2. Use long division to divide polynomials.
3. Divide polynomial functions.

In Section 5.1, we learned the quotient rule for exponents. Although we did not say so at that time, we used the quotient rule to divide monomials, as shown here.

\[
\frac{28x^5}{4x^2} = \frac{28}{4} x^{5-2} = 7x^3
\]

In this section, we use the quotient rule for exponents and divide polynomials with more terms.

Rule

Division of a Polynomial by a Monomial

If \( a, b, \) and \( c \) are real numbers, variables, or expressions with \( c \neq 0 \), then

\[
\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}.
\]

We can apply this rule to \( \frac{28x^5 + 20x^3}{4x^2} \).

\[
\frac{28x^5 + 20x^3}{4x^2} = \frac{28x^5}{4x^2} + \frac{20x^3}{4x^2} = 7x^{5-2} + 5x^{3-2} = 7x^3 + 5x
\]

Note: We now have a sum of monomial divisions, which we can simplify separately.

This illustration suggests the following procedure.

Procedure

Dividing a Polynomial by a Monomial

To divide a polynomial by a monomial, divide each term in the polynomial by the monomial.
Example 1  Divide.

a. \( \frac{12u^6 - 18u^4 + 42u^2}{6u^2} \)

SOLUTION

\[
\frac{12u^6 - 18u^4 + 42u^2}{6u^2} = \frac{12u^6}{6u^2} - \frac{18u^4}{6u^2} + \frac{42u^2}{6u^2} \\
= 2u^4 - 3u^2 + 7
\]

Divide each term in the polynomial by the monomial.

b. \((32x^5y^3 - 24x^4y^2 - 2xy) + 8x^2y\)

SOLUTION

\[
(32x^5y^3 - 24x^4y^2 - 2xy) + 8x^2y = \frac{32x^5y^3 - 24x^4y^2 - 2xy}{8x^2y} \\
= \frac{32x^5y^3}{8x^2y} - \frac{24x^4y^2}{8x^2y} - \frac{2xy}{8x^2y} \\
= 4x^3y^2 - 3x^2y - \frac{1}{4x}
\]

Divide each term in the polynomial by the monomial.

Note: Using the quotient rule, we have \( \frac{2xy}{8x^2y} = \frac{1}{4}x^{-1} = \frac{1}{4x} \).
Your Turn 1  Divide.

a. \[ \frac{54x^5 + 42x^4 - 24x^3}{6x^3} \]

b. \( (12a^2b^5 - 20a^4b + 6a^2) + 4a^2b \)

\[ \frac{54x^5}{6x^3} + \frac{42x^4}{6x^3} - \frac{24x^3}{6x^3} \]

\[ 9x^2 + 7x - 4 \]