### 3.5 Dividing Decimals

#### 1 Dividing a Decimal by a Whole Number

When you divide a decimal by a whole number, place the decimal point for the quotient directly above the decimal point in the dividend. Then divide as if the numbers were whole numbers.

To divide 26.8 by 4, we place the decimal point of our answer (the quotient) directly above the decimal point in the dividend.

\[
\begin{array}{c|c}
 & 26.8 \\
\hline
4 & \_ \\
\hline
& 6.7
\end{array}
\]

The decimal points are aligned, one above the other.

Then we divide as if we were dividing whole numbers.

\[
\begin{array}{c|c}
 & 6.7 \\
\hline
4 & 26.8 \\
\hline
& 24 \\
\hline
& 28 \\
\hline
& 28 \\
\hline
& 0
\end{array}
\]

The quotient is 6.7.

The quotient to a problem may have all digits to the right of the decimal point. In some cases you will have to put a zero in the quotient as a "place holder." Let's divide 0.268 by 4.

\[
\begin{array}{c|c}
 & 0.678 \\
\hline
4 & 0.268 \\
\hline
& 24 \\
\hline
& 28 \\
\hline
& 28 \\
\hline
& 0
\end{array}
\]

Note that we must have a zero after the decimal point in 0.067.
Practice Problem 1

Divide.

(a) \( \frac{1.806}{7} \)\( \div \)\( \frac{8}{25} \)

(b) \( \frac{0.0928}{16} \)

\[
\begin{align*}
1.806 & \div 7 \\
7 & \longdiv{1.806} \\
-14 & \\
\hline
40 & \\
-35 & \\
\hline
56 & \\
56 & \\
\hline
0 & \\
\end{align*}
\]

\[
\begin{align*}
0.0928 & \div 16 \\
16 & \longdiv{0.0928} \\
-80 & \\
\hline
128 & \\
-128 & \\
\hline
0 & \\
\end{align*}
\]
**EXAMPLE 2**  Divide and round the quotient to the nearest thousandth.

\[ 12.67 \div 39 \]

**Solution**  We will carry out our division to the ten-thousandths place. Then we will round our answer to the nearest thousandth.

\[
\begin{array}{c}
0.3248 \\
39 \overline{)12.6700} \\
11.7 \\
97 \\
78 \\
190 \\
156 \\
340 \\
312 \\
28
\end{array}
\]

Two extra zeros are written here to carry out the division to the required place.

Note that the remainder is not zero.

Now we round 0.3248 to \(0.325\). The answer is rounded to the nearest thousandth.

**Practice Problem 2**  Divide and round the quotient to the nearest hundredth. \(23.82 \div 46\)
EXAMPLE 3  Maria paid $5.92 for 16 pounds of tomatoes. How much did she pay per pound?

Solution  The cost of one pound of tomatoes equals the total cost, $5.92, divided by 16 pounds. Thus we will divide.

\[
\begin{array}{c}
\text{0.37} \\
16)5.92 \\
\underline{4.8} \\
11 \underline{2} \\
11 \underline{2} \\
\underline{0}
\end{array}
\]

Maria paid $0.37 per pound for the tomatoes.

Practice Problem 3  Won Lin will pay off his auto loan for $3538.75 over 19 months. If the monthly payments are equal, how much will he pay each month?

Mathematics Blueprint for Problem Solving

<table>
<thead>
<tr>
<th>Gather the Facts</th>
<th>What Am I Asked to Do?</th>
<th>How Do I Proceed?</th>
<th>Key Points to Remember</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own $3538.75</td>
<td>Make 19 Equal Payments</td>
<td>Divide 3538.75 ( \div 19 )</td>
<td>( \frac{19}{2} \times \frac{5}{6} )</td>
</tr>
<tr>
<td></td>
<td>First Payment</td>
<td>186.25</td>
<td>( \frac{19}{5} \times \frac{9.5}{152} )</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
186.25
\end{array}
\]
2 Dividing a Decimal by a Decimal

When the divisor is not a whole number, we can convert the division problem to an equivalent problem that has a whole number as a divisor. Think about the reasons why this procedure will work. We will ask you about it after you study Examples 4 and 5.

DIVIDING A DECIMAL BY A DECIMAL

1. Make the divisor a whole number by moving the decimal point to the right. Mark that position with a caret (\^). Count the number of places the decimal point moved.
2. Move the decimal point in the dividend to the right the same number of places. Mark that position with a caret.
3. Place the decimal point of your answer directly above the caret marking the decimal point of the dividend.
4. Divide as with whole numbers.

Practice Problem 4

Divide.

(a) \( \frac{0.09}{0.1008} \)  
(b) \( 1.702 \div 0.037 \)

\[
\begin{align*}
\frac{0.09}{0.1008} &= \frac{9}{1008} = \frac{1}{112} \\
\frac{1.702}{0.037} &= 46.2
\end{align*}
\]
**TO THINK ABOUT: The Multiplicative Identity** Why do we move the decimal point to the right in the divisor and the dividend? What rule allows us to do this? How do we know the answer will be valid? We are actually using the property that multiplication of a fraction by 1 leaves the fraction unchanged. This is called the *multiplicative identity*. Let us examine Example 4(b) again. We will write $1.352 \div 0.026$ as a fraction.

\[
\frac{1.352}{0.026} \times 1
\]

Multiplication of a fraction by 1 does not change the value of the fraction.

\[
= \frac{1.352}{0.026} \times \frac{1000}{1000}
\]

We know that $\frac{1000}{1000} = 1$.

\[
= \frac{1352}{26}
\]

Multiplication by 1000 can be done by moving the decimal point three places to the right.

\[
= 52
\]

Divide the whole numbers.

Thus in Example 4(b) when we moved the decimal point three places to the right in the divisor and the dividend, we were actually creating an equivalent fraction where the numerator and the denominator of the original fraction were multiplied by 1000.
Practice Problem 5  
Divide.

(a) \(1.8 \overline{)0.0414}\)

\[
\begin{array}{c|c}
1.8 & 0.0414 \\
\hline
& 0.023 \\
-36 & \ \\
\hline
54 & \ \\
-54 & \ \\
\hline
& 0
\end{array}
\]

(b) \(0.0036 \overline{)8.316}\)

\[
\begin{array}{c|c}
& 2.320 \\
0.0036 & 8.3160 \\
\hline
& 0.72 \\
-72 & \ \\
\hline
& 0.0 \\
-0.0 & \ \\
\hline
& 0
\end{array}
\]

\[
\]
(a) Find \(3.8 \div 521.6\) rounded to the nearest tenth.

\[
\begin{array}{c}
3.8\
\times 2\
\hline
76
\end{array}
\]

\[
\begin{array}{c}
3.8\
\times 5\
\hline
190
\end{array}
\]

\[
\begin{array}{c}
3.8\
\times 4\
\hline
152
\end{array}
\]

\[
\begin{array}{c}
3.8\
\times 3\
\hline
114
\end{array}
\]

\[
\begin{array}{c}
63.8\
\times 8\
\hline
504
\end{array}
\]

\[
\begin{array}{c}
137.26\
-38\hline
-100
\end{array}
\]

\[
\begin{array}{c}
27.6\
-26.8\hline
-0.8
\end{array}
\]

\[
\begin{array}{c}
2.48\
-2.28\hline
0.20
\end{array}
\]

\[
\begin{array}{c}
0.12
\end{array}
\]

Round 0.12 to the nearest tenth gives 0.1.
Practice Problem 6

(b) Find $8.05 \div 0.17$ rounded to the nearest thousandth.

\[
\begin{array}{c}
8.05 \div 0.17 \\
\hline
805 \div 17.0000 \\
-16.10 \\
\hline
90.0 \\
86.5 \\
\hline
9.5 \\
8.5 \\
\hline
1.5
\end{array}
\]

\[0.021\]
**Practice Problem 7** Sarah rented a large truck to move to Boston. She drove 454.4 miles yesterday. She used 28.5 gallons of gas on the trip. How many miles per gallon did the rental truck get? Round to the nearest tenth.

**Mathematics Blueprint for Problem Solving**

<table>
<thead>
<tr>
<th>Gather the Facts</th>
<th>What Am I Asked to Do?</th>
<th>How Do I Proceed?</th>
<th>Key Points to Remember</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Practice Problem 9** Use the accompanying bar graph to find the average level of sulfur dioxide for the three years: 1985, 1990, and 1995. By how much does the three-year average differ from the five-year average?