5.2 Changing Between Percents, Decimals, and Fractions

1 Changing a Percent to a Fraction

By using the definition of percent, we can write any percent as a fraction whose denominator is 100. Thus when we change a percent to a fraction, we remove the percent symbol and write the number over 100. To write a number over 100 means that we are dividing by 100. If possible, we then simplify the fraction.

**EXAMPLE 1** Write as a fraction in simplest form.

<table>
<thead>
<tr>
<th></th>
<th>(a) 37%</th>
<th>(b) 75%</th>
<th>(c) 2%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution</strong></td>
<td>( \frac{37}{100} = .37 )</td>
<td>( \frac{75}{100} = \frac{3}{4} = .75 )</td>
<td>( \frac{2}{100} = \frac{1}{50} = .02 )</td>
</tr>
</tbody>
</table>

**Practice Problem 1** Write as a fraction in simplest form.

<table>
<thead>
<tr>
<th></th>
<th>(a) 71%</th>
<th>(b) 25%</th>
<th>(c) 8%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Practice Problem 1</strong></td>
<td>( \frac{71}{100} = .71 )</td>
<td>( \frac{25}{100} = \frac{1}{4} = .25 )</td>
<td>( \frac{8}{100} = \frac{4}{50} = \frac{2}{25} = .08 )</td>
</tr>
</tbody>
</table>
EXAMPLE 2 Write as a fraction in simplest form.

(a) 43.5%  

Solution

(a) $43.5\% = 0.435$  

$= \frac{435}{1000}$  

$= \frac{87}{200}$  

(b) $36.75\% = 0.3675 = \frac{3675}{10,000} = \frac{147}{400}$

Practice Problem 2 Write as a fraction in simplest form.

(a) 8.4%  

(b) 28.5%
If the percent is greater than 100%, the simplified fraction is usually changed to a mixed number.

**EXAMPLE 3**  Write as a mixed number.

(a) 225%  

\[225\% = 2.25 = 2 \frac{25}{100} = 2 \frac{1}{4}\]

(b) 138%  

\[138\% = 1.38 = 1 \frac{38}{100} = 1 \frac{19}{50}\]

**Practice Problem 3**  Write as a mixed number.

(a) 170%  

\[170\% = 1.7 = 1 \frac{70}{100} = 1 \frac{7}{10}\]

(b) 288%  

\[288\% = 2.88 = 2 \frac{88}{100} = 2 \frac{22}{25}\]
EXAMPLE 4  Convert $3\frac{3}{8}\%$ to a fraction in simplest form.

**Solution**

\[
3\frac{3}{8}\% = \frac{\frac{3}{8}}{100}
\]

Change the percent to a fraction.

\[
= \frac{3}{8} \div \frac{100}{1}
\]

Write the division horizontally. $\frac{3}{8}$ means $\frac{3}{8}$ divided by 100.

\[
= \frac{27}{8} \div \frac{100}{1}
\]

Write $3\frac{3}{8}$ as an improper fraction.

\[
= \frac{27}{8} \times \frac{100}{1}
\]

Use the definition of division of fractions.

\[
= \frac{27}{800}
\]

Simplify.

**Practice Problem 4**  Convert $7\frac{5}{8}\%$ to a fraction in simplest form.

\[
7\frac{5}{8}\% = \frac{\frac{7}{8}}{100} = \frac{7}{8} \div 100
\]

\[
= \frac{61}{8} \div \frac{100}{1}
\]

\[
= \frac{61}{8} \times \frac{1}{100} = \frac{61}{800}
\]

\[
61 \div 800 = 0.07625
\]

Decimal

\[
5\frac{2}{9}\% = \frac{\frac{5}{9}}{100} = \frac{\frac{5}{9}}{\frac{100}{1}} = \frac{5}{9} \div \frac{100}{1}
\]

\[
= \frac{37}{200}
\]

\[
\frac{37}{200} = 0.185714
\]

\[
= \frac{37}{200}
\]
Practice Problem 5: In the fiscal 2007 budget of the United States, approximately $20\frac{7}{8}\%$ was designated for social security. (Source: U.S. Office of Management and Budget.) Write this percent as a fraction.
Certain percents occur very often, especially in money matters. Here are some common equivalents that you may already know. If not, be sure to memorize them.

\[
25\% = \frac{1}{4} \quad 33\frac{1}{3}\% = \frac{1}{3} \quad 10\% = \frac{1}{10}
\]

\[
50\% = \frac{1}{2} \quad 66\frac{2}{3}\% = \frac{2}{3}
\]

\[
75\% = \frac{3}{4}
\]
2 Changing a Fraction to a Percent

A convenient way to change a fraction to a percent is to write the fraction in decimal form first and then convert the decimal to a percent.

**EXAMPLE 6** Write \( \frac{3}{8} \) as a percent.

**Solution** We see that \( \frac{3}{8} = 0.375 \) by calculating \( 3 \div 8 \).

\[
\begin{array}{r}
0.375 \\
\hline
3.000 \\
8 \\
\hline
24 \\
24 \\
\hline
0 \\
60 \\
\hline
56 \\
56 \\
\hline
0 \\
40 \\
\hline
40 \\
40 \\
\hline
0
\end{array}
\]

Thus \( \frac{3}{8} = 0.375 = 37.5\% \).

**Practice Problem 6** Write \( \frac{5}{8} \) as a percent.

\( \frac{5}{8} = 0.625 = 62.5\% \)

\( 5 \div 8 = \)
EXAMPLE 7  Write as a percent.

(a) \( \frac{7}{40} \)  (b) \( \frac{39}{50} \)

Solution

(a) \( \frac{7}{40} = 0.175 = 17.5\% \)  (b) \( \frac{39}{50} = 0.78 = 78\% \)

Practice Problem 7  Write as a percent.

(a) \( \frac{21}{25} \)  (b) \( \frac{7}{16} \)

\[ \frac{21}{25} = .84 = 84\% \]

\[ \frac{7}{16} = 0.4375 = \frac{4375}{10000} \]

\[ 21 \div 25 = \]

\[ 21 \div 25 = \]

\[ 16 \quad \overline{17.000} \]
Changing some fractions to decimal form results in infinitely repeating decimals. In such cases, we usually round to the nearest hundredth of a percent.

**EXAMPLE 8** Write as a percent. Round to the nearest hundredth of a percent.

(a) \( \frac{1}{6} = .1666\ldots \)

\[
\begin{array}{c}
0.1666 \\
6)1.0000 \\
6 \\
40 \\
36 \\
40 \\
36 \\
40 \\
36 \\
4 \\
\end{array}
\]

We will need a four-place decimal so that we will obtain a percent to the nearest hundredth. If we round the decimal to the nearest thousandth, we have \( \frac{1}{6} \approx 0.1667 \). If we change this to a percent, we have \( \frac{1}{6} \approx 16.67\% \).

This is correct to the nearest hundredth of a percent.

(b) By calculating \( 15 \div 33 \), we see that \( \frac{15}{33} = 0.45454545\ldots \). We will need a four-place decimal so that we will obtain a percent to the nearest hundredth. If we round to the nearest thousandth, we have \( \frac{15}{33} \approx 0.4545 = 45.45\% \).
Practice Problem 8 Write as a percent. Round to the nearest hundredth of a percent.

<table>
<thead>
<tr>
<th>(a)</th>
<th>[ \frac{7}{9} ]</th>
<th>(b)</th>
<th>[ \frac{19}{30} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[ \approx 77.78% ]</td>
<td></td>
<td>[ \approx 63.33% ]</td>
</tr>
</tbody>
</table>
EXAMPLE 9  Express $\frac{11}{12}$ as a percent containing a fraction.

Solution  We will stop the division after two steps and write the remainder in fraction form.

\[
\begin{array}{c|cc}
 & 0.91 \\
\hline
12 & 11.00 \\
& 108 \\
& 20 \\
& 12 \\
& 8 \\
\end{array}
\]

This division tells us that we can write

\[
\frac{11}{12} \text{ as } 0.91 \frac{8}{12} \text{ or } 0.91 \frac{2}{3}.
\]

We now have a decimal with a fraction. When we express this decimal as a percent, we move the decimal point two places to the right. We do not write the decimal point in front of the fraction.

\[
0.91 \frac{2}{3} = 91 \frac{2}{3}\%.
\]

Note that our answer in Example 9 is an exact answer. We have not rounded off or approximated in any way.

Practice Problem 9  Express $\frac{7}{12}$ as a percent containing a fraction.
Changing a Percent, a Decimal, or a Fraction to Equivalent Forms

We have seen so far that a fraction, a decimal, and a percent are three different forms (notations) for the same number. We can illustrate this in a chart.
Practice Problem 10: Complete the following table of equivalent notations. Round decimals to the nearest ten-thousandth. Round percents to the nearest hundredth of a percent.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{23}{99} )</td>
<td>( 0.2323 )</td>
<td>( 23.23% )</td>
</tr>
<tr>
<td>( \frac{129}{250} )</td>
<td>( 0.516 )</td>
<td>( 51.6% )</td>
</tr>
<tr>
<td>( \frac{387}{1000} )</td>
<td>( 0.387 )</td>
<td>( \frac{387}{5} % = 77.4% )</td>
</tr>
</tbody>
</table>

\[
\frac{23}{99} = 0.232323 \approx 0.2323
\]

\[
0.516 = \frac{516}{1000} = \frac{129}{250}
\]

\[
\frac{4}{5} = 0.8\% = 0.8
\]
Write as a fraction or as a mixed number.

3. 6%  
4. 8%  
5. 33%  

8. 35%  
9. 75%  
10. 25%  

13. 9.5%  
14. 6.5%  
15. 22.5%  

18. 12.2%  
19. 71.25%  
20. 38.75%  

23. 340%  
24. 420%  
25. 1200%  

28. $\frac{43}{5}\%$  
29. $12\frac{1}{2}\%$  
30. $37\frac{1}{2}\%$
34. **Crime Rates** Between 1996 and 2005, the number of property crimes in the United States decreased by 22.9%. Write the percent as a fraction. *(Source: www.ojp.usdoj.gov)*
Write as a percent. Round to the nearest hundredth of a percent when necessary.

<table>
<thead>
<tr>
<th>37. ( \frac{3}{4} )</th>
<th>38. ( \frac{1}{4} )</th>
<th>39. ( \frac{7}{10} )</th>
<th>40. ( \frac{9}{10} )</th>
<th>41. ( \frac{7}{20} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>43. ( \frac{18}{25} )</td>
<td>44. ( \frac{22}{25} )</td>
<td>45. ( \frac{11}{40} )</td>
<td>46. ( \frac{13}{40} )</td>
<td>47. ( \frac{18}{5} )</td>
</tr>
<tr>
<td>49. ( \frac{21}{2} )</td>
<td>50. ( \frac{33}{4} )</td>
<td>51. ( \frac{41}{8} )</td>
<td>52. ( \frac{25}{8} )</td>
<td>53. ( \frac{1}{3} )</td>
</tr>
<tr>
<td>55. ( \frac{5}{12} )</td>
<td>56. ( \frac{8}{15} )</td>
<td>57. ( \frac{17}{4} )</td>
<td>58. ( \frac{12}{5} )</td>
<td>59. ( \frac{26}{50} )</td>
</tr>
</tbody>
</table>
Express as a percent containing a fraction. (See Example 9.)

65. $\frac{3}{8}$  
66. $\frac{5}{8}$  
67. $\frac{3}{40}$

69. $\frac{4}{15}$  
70. $\frac{11}{15}$  
71. $\frac{2}{9}$
<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{12}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{7}{9}$</td>
<td></td>
<td>$2\frac{5}{8}%$</td>
</tr>
<tr>
<td>0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.085</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>