1.6 Exponents and the Order of Operations

Evaluating Expressions with Whole-Number Exponents

Sometimes a simple math idea comes “disguised” in technical language. For example, an exponent is just a “shorthand” number that saves writing multiplication of the same numbers.

\[ 10^3 \quad \text{means} \quad 10 \times 10 \times 10 \]

(which takes longer to write).

The product \( 5 \times 5 \) can be written as \( 5^2 \). The small number 2 is called the exponent. The exponent tells us how many factors are in the multiplication. The number 5 is called the base. The base is the number that is multiplied.

\[ 3 \times 3 \times 3 \times 3 = 3^4 \]

In \( 3^4 \) the base is 3 and the exponent is 4. (The 4 is sometimes called the superscript.) \( 3^4 \) is read as “three to the fourth power.”

\[(a) \quad 15 \times 15 \times 15 = 15^3 \quad (b) \quad 7 \times 7 \times 7 \times 7 \times 7 = 7^5\]

Practice Problem 1

Write each product in exponent form.

(a) \( 12 \times 12 \times 12 \times 12 \) \quad (b) \( 2 \times 2 \times 2 \times 2 \times 2 \times 2 \)

\[ 12^4 \quad 2^6 = 64 \]

Practice Problem 2

Find the value of each expression.

(a) \( 12^2 \) \quad (b) \( 6^3 \) \quad (c) \( 2^6 \) \quad (d) \( 1^{10} = 1 \)

\[ 12 \times 12 = 144 \quad 6 \times 6 \times 6 = 216 \quad \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2} \]

\[ 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \]

10 Times
If a whole number does not have a visible exponent, the exponent is understood to be 1. Thus

\[ 3 = 3^1 \quad \text{and} \quad 10 = 10^1. \]

Large numbers are often expressed as a power of 10.

\[
\begin{align*}
10^1 &= 10 = 1 \text{ ten} & 10^4 &= 10,000 = 1 \text{ ten thousand} \\
10^2 &= 100 = 1 \text{ hundred} & 10^5 &= 100,000 = 1 \text{ hundred thousand} \\
10^3 &= 1000 = 1 \text{ thousand} & 10^6 &= 1,000,000 = 1 \text{ million}
\end{align*}
\]

What does it mean to have an exponent of zero? What is \(10^0\)? Any whole number that is not zero can be raised to the zero power. The result is 1. Thus \(10^0 = 1, 3^0 = 1, 5^0 = 1\), and so on. Why is this? Let's reexamine the powers of 10. As we go down one line at a time, notice the pattern that occurs.

As we move down one line, we decrease the exponent by 1.

\[
\begin{align*}
10^5 &= 100,000 \quad \rightarrow 10^4 &= 10,000 \quad \rightarrow 10^3 &= 1000 \quad \rightarrow 10^2 &= 100 \quad \rightarrow 10^1 &= 10 \quad \rightarrow 10^0 &= 1
\end{align*}
\]

As we move down one line, we divide the previous number by 10.

Therefore, we present the following definition.

For any whole number \(a\) other than zero, \(a^0 = 1\).
If numbers with exponents are added to other numbers, it is first necessary to **evaluate**, or find the value of, the number that is raised to a power. Then we may combine the results with another number.

**Practice Problem 3**
Find the value of each expression.

(a) \(7^3 + 8^2\)  
(b) \(9^2 + 6^0\)  
(c) \(5^4 + 5\)

\[
\begin{align*}
7^3 + 8^2 &= 401 \\
&= 343 + 64 \\
&= 64 + 49 \\
&= 169 + 64 \\
&= 343 - 901 \\
&= 5^{14} + 5 \\
&= 5 \times 5 \times 5 \times 5 \\
&= 625 + 5 \\
&= \frac{125}{5} \\
&= 625
\end{align*}
\]
Performing Several Arithmetic Operations in the Proper Order

Sometimes the order in which we do things is not important. The order in which chefs hang up their pots and pans probably does not matter. The order in which they add and mix the elements in preparing food, however, makes all the difference in the world! If various cooks follow a recipe, though, they will get similar results. The recipe assures that the results will be consistent. It shows the order of operations.

In mathematics the order of operations is a list of priorities for working with the numbers in computational problems. This mathematical "recipe" tells how to handle certain indefinite computations. For example, how does a person find the value of $5 + 3 \times 2$?

A problem such as $5 + 3 \times 2$ sometimes causes students difficulty. Some people think $(5 + 3) \times 2 = 8 \times 2 = 16$. Some people think $5 + (3 \times 2) = 5 + 6 = 11$. Only one answer is right, 11. To obtain the right answer, follow the steps outlined in the following box.

**ORDER OF OPERATIONS**

In the absence of grouping symbols:

- **Do first**
  - 1. Simplify any expressions with exponents.
  - 2. Multiply or divide from left to right.

- **Do last**
  - 3. Add or subtract from left to right.
Practice Problem 5: Evaluate. \(37 - 20 + 5 + 2 - 3 \times 4\)

\[37 - 4 + 2 - 3 \times 4\]

\[37 - 4 + 2 - 12\]

\[33 + 2 - 12\]

\[33 - 12\]

\[21\]

Practice Problem 4: Evaluate. \(7 + 4^3 \times 3\)

\[7 + 64 \times 3\]

\[7 + 192\]

\[199\]
Practice Problem 6

Evaluate: \( 4^3 - 2 + 3^2 \)

\[
4^3 - 2 + 3^2 \\
64 - 2 + 9 \\
62 + 9 \\
\boxed{71}
\]
You can change the order in which you compute by using grouping symbols. Place the numbers you want to calculate first within parentheses. This tells you to do those calculations first.

**ORDER OF OPERATIONS**

With grouping symbols:

Do first 1. Perform operations inside parentheses.

2. Simplify any expressions with exponents.

3. Multiply or divide from left to right.

Do last 4. Add or subtract from left to right.

**Practice Problem 7**

Evaluate: \((17 + 7) ÷ 6 × 2 + 7 × 3 - 4\)
Evaluate: \(5^2 - 6 \div 2 + 3^3 + 7 \times (12 - 10)\)

\[
\begin{align*}
5^2 & - 6 \div 2 + 3^3 + 7 \times 2 \\
25 & - 6 \div 2 + 27 + 14 \\
25 & - 3 + 81 + 14 \\
= 117
\end{align*}
\]