2.1 Understanding Fractions

Using a Fraction to Represent Part of a Whole

In Chapter 1 we studied whole numbers. In this chapter we will study a fractional part of a whole number. One way to represent parts of a whole is with fractions. The word fraction (like the word fracture) suggests that something is being broken. In mathematics, fractions represent the part that is “broken off” from a whole. The whole can be a single object (like a whole pie) or a group (the employees of a company). Here are some examples.

Single object

The whole is the pie on the left. The fraction $\frac{1}{3}$ represents the shaded part of the pie, $\frac{1}{3}$ of 3 pieces. $\frac{1}{3}$ is read “one-third.”

A group: ACE company employs 150 men, 200 women.

\[
\frac{150}{350}
\]

The whole is the company of 350 people (150 men plus 200 women). The fraction $\frac{150}{350}$ represents that part of the company consisting of men.

When we say “$\frac{3}{8}$ of a pizza has been eaten,” we mean 3 of 8 equal parts of a pizza have been eaten. (See the figure.) When we write the fraction $\frac{3}{8}$, the number on the top, 3, is the numerator, and the number on the bottom, 8, is the denominator.

When we say, “$\frac{2}{3}$ of the marbles are red,” we mean 2 marbles out of a total of 3 are red marbles.

Part we are interested in $\rightarrow$ numerator
Total number in the group $\rightarrow$ denominator
We can also think of a fraction as a division problem.

\[
\frac{1}{3} = 1 \div 3 \quad \text{and} \quad 1 \div 3 = \frac{1}{3}
\]

The division way of looking at fractions asks the question:

What is the result of dividing one whole into three equal parts?

Thus we can say the fraction \(\frac{a}{b}\) means the same as \(a \div b\). However, special care must be taken with the number 0.

Suppose that we had four equal parts and we wanted to take none of them. We would want \(\frac{0}{4}\) of the parts. Since \(\frac{0}{4} = 0 \div 4 = 0\), we see that \(0 = 0\). Any fraction with a 0 numerator equals zero.

\[
\begin{align*}
\frac{0}{8} &= 0 \\
\frac{0}{5} &= 0 \\
\frac{0}{13} &= 0
\end{align*}
\]

What happens when zero is in the denominator? \(\frac{4}{0}\) means 4 out of 0 parts. Taking 4 out of 0 does not make sense. We say \(\frac{4}{0}\) is undefined.

\[
\begin{align*}
\frac{3}{0}, & \quad \frac{7}{0}, & \quad \frac{4}{0} \quad \text{are undefined.} \\
\frac{6}{2} &= 3 \\
2 \cdot 3 &= 6
\end{align*}
\]

We cannot have a fraction with 0 in the denominator. Since \(\frac{4}{0} = 4 \div 0\), we say division by zero is undefined. We cannot divide by 0.

\[
\begin{align*}
\frac{0}{2} &= 0 & \text{Because} & \quad 2 \cdot 0 = 0 \\
\frac{2}{0} &= \text{undefined} & \text{Because} & \quad ? \times 0 = 2
\end{align*}
\]
Drawing a Sketch to Illustrate a Fraction

Drawing a sketch of a mathematical situation is a powerful problem-solving technique. The picture often reveals information not always apparent in the words.

**Practice Problem 2** Draw a sketch to illustrate.

(a) $\frac{4}{5}$ of an object

(b) $\frac{3}{7}$ of a group
Recall these facts about division problems involving the number 1 and the number 0.

**DIVISION INVOLVING THE NUMBER 1 AND THE NUMBER 0**

1. Any nonzero number divided by itself is 1.
   \[
   \frac{7}{7} = 1
   \]

2. Any number divided by 1 remains unchanged.
   \[
   \frac{29}{1} = 29
   \]

3. Zero may be divided by any nonzero number; the result is always zero.
   \[
   \frac{0}{4} = 0
   \]

4. Division by zero is undefined.
   \[
   \frac{3}{0} \text{ is undefined}
   \]
Using Fractions to Represent Real-Life Situations

Many real-life situations can be described using fractions.

Practice Problem 4  An inspector found that one out of seven belts was defective. She also found that two out of nine shirts were defective. Write a fraction that describes what part of all the objects examined were defective.

\[ \frac{1 + 2}{9 + 7} = \frac{3}{16} \]

Practice Problem 3  Use a fraction to describe each situation.

(a) 9 out of the 17 players on the basketball team are on the dean’s list.
(b) The senior class has 382 men and 351 women. Describe the part of the class consisting of men.
(c) John needed seven-eighths of a yard of material.
Name the numerator and the denominator in each fraction.

5. $\frac{3}{5}$  
6. $\frac{9}{11}$  
7. $\frac{7}{8}$  
8. $\frac{9}{10}$

In exercises 11–30, use a fraction to represent the shaded part of the object or the shaded portion of the set of objects.

11.  
12. $\frac{1}{2}$  
13.  
14.  
15.  
16.  
17.  
18. $\frac{3}{8}$  
19.  
20.  
21.  
22.  

6
Draw a sketch to illustrate each fractional part.

31. \( \frac{1}{5} \) of an object  
32. \( \frac{3}{7} \) of an object  
34. \( \frac{5}{12} \) of an object  
35. \( \frac{7}{10} \) of an object

38. **Personal Finance** Miguel bought a notebook with a total purchase price of 98¢. Of this amount, 7¢ was sales tax. What fractional part of the total purchase price was sales tax?

40. **Personal Finance** Jillian earned $165 over the weekend at her waitressing job. She used $48 of it to repay a loan to her sister. What fractional part of her earnings did Jillian use to repay her sister?

42. **Education** Bridgeton Community College has 78 full-time instructors and 31 part-time instructors. What fractional part of the faculty are part time?

44. **Animal Shelters** At the local animal shelter there are 12 puppies, 25 adult dogs, 14 kittens, and 31 adult cats. What fractional part of the animals are either puppies or adult dogs?