Section 2.2

216

$\sqrt{216} = 2.16$  
$1, 2, 3, 5, 7, 11$

2

108

54

27

9

3

3

3

$\frac{3}{2}$

Prime Factorization

$2 \cdot 3^3$
Cross Products Equal

\[
\frac{1}{2} \times \frac{3}{16}
\]

\[
2 \times 8 = 1 \times 16
\]

16 = 16
2.3 Converting Between Improper Fractions and Mixed Numbers

1. Changing a Mixed Number to an Improper Fraction

We have names for different kinds of fractions. If the value of a fraction is less than 1, we say the fraction is proper.

\[
\frac{3}{5}, \frac{5}{7}, \frac{1}{8}
\]

are called proper fractions.

Notice that the numerator is less than the denominator. If the numerator is less than the denominator, the fraction is a proper fraction.

If the value of a fraction is greater than or equal to 1, the quantity can be written as an improper fraction or as a mixed number.

Suppose that we have 1 whole pizza and \( \frac{1}{6} \) of a pizza. We could write this as \( 1 \frac{1}{6} \). \( 1 \frac{1}{6} \) is called a mixed number. A mixed number is the sum of a whole number greater than zero and a proper fraction. The notation \( 1 \frac{1}{6} \) actually means \( 1 + \frac{1}{6} \). The plus sign is not usually shown.

Another way of writing \( 1 \frac{1}{6} \) pizza is to write \( \frac{7}{6} \) pizza. \( \frac{7}{6} \) is called an improper fraction. Notice that the numerator is greater than the denominator. If the numerator is greater than or equal to the denominator, the fraction is an improper fraction.

\[
\frac{7}{6} = \frac{11}{8} = 1 \frac{3}{8}
\]

Mixed Number
The following chart will help you visualize these different fractions and their names.

Because in some cases improper fractions are easier to add, subtract, multiply, and divide than mixed numbers, we often change mixed numbers to improper fractions when we perform calculations with them.

<table>
<thead>
<tr>
<th>Value Less Than 1</th>
<th>Value Equal To 1</th>
<th>Value Greater Than 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper Fraction</td>
<td>Improper Fraction</td>
<td>Improper Fraction or Mixed Number</td>
</tr>
<tr>
<td><img src="image1" alt="Fraction" /></td>
<td><img src="image2" alt="Fraction" /></td>
<td><img src="image3" alt="Fraction" /></td>
</tr>
<tr>
<td><img src="image4" alt="Fraction" /></td>
<td><img src="image5" alt="Fraction" /></td>
<td><img src="image6" alt="Fraction" /></td>
</tr>
<tr>
<td><img src="image7" alt="Fraction" /></td>
<td><img src="image8" alt="Fraction" /></td>
<td><img src="image9" alt="Fraction" /></td>
</tr>
</tbody>
</table>
CHANGING A MIXED NUMBER TO AN IMPROPER FRACTION

1. Multiply the whole number by the denominator of the fraction.
2. Add the numerator of the fraction to the product found in step 1.
3. Write the sum found in step 2 over the denominator of the fraction.

**Example 1**

Change each mixed number to an improper fraction.

(a) \(3 \frac{2}{5}\)  
(b) \(5 \frac{4}{9}\)  
(c) \(18 \frac{3}{5}\)

**Solution**

(a) \(\frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{17}{5}\)

(b) \(\frac{4}{9} = \frac{5 \times 9 + 4}{9} = \frac{49}{9}\)

**Practice Problem 1**

Change the mixed numbers to improper fractions.

(a) \(4 \frac{3}{7}\)  
(b) \(6 \frac{2}{3}\)  
(c) \(19 \frac{4}{7}\)

(a) \(4 \times 7 = \frac{28}{7} = 31\)  
(b) \(6 \times 3 = \frac{18}{3} + \frac{2}{3} = \frac{20}{3}\)  
(c) \(19 \times 7 = 133\)  
\(\frac{4}{7} = \frac{132}{7}\)
CHANGING AN IMPROPER FRACTION TO A MIXED NUMBER

1. Divide the numerator by the denominator.
2. Write the quotient followed by the fraction with the remainder over the denominator.
   
   \[
   \frac{\text{quotient}}{\text{remainder}} = \frac{\text{remainder}}{\text{denominator}}
   \]

**Example 2** Write each improper fraction as a mixed number.

(a) \( \frac{13}{5} \)  
(b) \( \frac{29}{7} \)  
(c) \( \frac{105}{31} \)  
(d) \( \frac{85}{17} \)  

**Solution**

(a) We divide the denominator 5 into 13.

\[
\begin{align*}
\text{quotient} & = \frac{13}{5} = 2 \cdot 5 + 3 \\
\text{remainder} & = 3 \\
\end{align*}
\]

The answer is in the form quotient \( \frac{\text{remainder}}{\text{denominator}} \).

Thus, \( \frac{13}{5} = 2 \frac{3}{5} \).

**Practice Problem 2** Write as a mixed number or a whole number.

(a) \( \frac{17}{4} \)  
(b) \( \frac{36}{5} \)  
(c) \( \frac{116}{27} \)  
(d) \( \frac{91}{13} \)  

\[
\begin{align*}
\frac{17}{4} &= 4 \frac{1}{4}  \\
\frac{36}{5} &= 7 \frac{1}{5}  \\
\frac{116}{27} &= 4 \frac{2}{27}  \\
\frac{91}{13} &= 7 \frac{2}{13}  \\
\end{align*}
\]
Reducing a Mixed Number or an Improper Fraction to Lowest Terms

Mixed numbers and improper fractions may need to be reduced if they are not in simplest form. Recall that we write the fraction in terms of prime factors. Then we look for common factors in the numerator and the denominator of the fraction. Then we divide the numerator and the denominator by the common factor.

**Example 3**

Reduce the improper fraction. \( \frac{22}{8} = \frac{4}{4} \)

**Solution**

\[
\frac{22}{8} = \frac{2 \times 11}{2 \times 2 \times 2} = \frac{11}{4}
\]

**Practice Problem 3**

Reduce the improper fraction. \( \frac{51}{15} \)

\[
\frac{51}{15} = \frac{17}{5}
\]

Reduced \( \frac{17}{5} \) -> Mixed # \( 3\frac{2}{5} \)

\[
5 \frac{3}{15} \rightarrow \frac{23}{15}
\]

\[
\frac{3}{15} \rightarrow \frac{21}{15}
\]
Practice Problem 4
Reduce the mixed number.

\[ \frac{16}{80} = \frac{8}{40} = \frac{4}{20} \]

\[ \frac{3 \frac{1}{5}}{20 \div 2} = \frac{3}{10} = \frac{3}{5} \]

Practice Problem 5
Reduce \( \frac{1001}{572} \) by first changing to a mixed number.
Change each mixed number to an improper fraction.

3. \(2 \frac{1}{3}\)  
4. \(2 \frac{3}{4}\)  
5. \(2 \frac{3}{7}\)  
6. \(3 \frac{3}{8}\)

Change each improper fraction to a mixed number or a whole number.

31. \(\frac{4}{3}\)  
32. \(\frac{13}{4}\)  
33. \(\frac{11}{4}\)  
34. \(\frac{9}{5}\)
Reduce each mixed number.

59. $5\frac{3}{6}$  
60. $4\frac{6}{8}$  
61. $4\frac{11}{66}$  
62. $3\frac{15}{90}$

Reduce each improper fraction.

65. $\frac{24}{6}$  
66. $\frac{36}{4}$  
67. $\frac{36}{15}$  
68. $\frac{63}{45}$
Change to a mixed number and reduce.

71. \[\frac{340}{126}\]  
72. \[\frac{390}{360}\]