1.3 Fractions

What is the difference between arithmetic and algebra? When doing arithmetic, all the quantities used in the calculations are known. In algebra, however, one or more of the quantities are unknown and must be found. Consider the following:

Mr. Piersma has 1 gallon of paint. In order to paint his bedroom, he needs 3 gallons of paint. How many additional gallons does he need?

This is an example of an algebraic problem. The *unknown* quantity is the number of additional gallons of paint needed. Mr. Piersma needs 2 more gallons of paint.

An understanding of decimal numbers (see Appendix A) and fractions is essential to success in algebra. You will need to know how to simplify a fraction and how to add, subtract, multiply, and divide fractions.

1 Learn Multiplication Symbols and Recognize Factors

In algebra we often use letters called **variables** to represent numbers. Letters commonly used as variables are \(x, y,\) and \(z,\) but other letters can be used as variables. Variables are usually shown in italics. So that we do not confuse the variable \(x\) with the multiplication sign \(\times\) we often use different notation to indicate multiplication.

**Multiplication Symbols**

If \(a\) and \(b\) represent any two mathematical quantities, then each of the following may be used to indicate the product of \(a\) and \(b\) ("\(a\) times \(b\)").

\[
ab \quad a \cdot b \quad a(b) \quad (a)b \quad (a)(b)
\]

**Examples**

<table>
<thead>
<tr>
<th>3 times 4</th>
<th>3 times (x)</th>
<th>(x) times (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>may be written:</td>
<td>may be written:</td>
<td>may be written:</td>
</tr>
<tr>
<td>(3(4))</td>
<td>(3(x))</td>
<td>(x(y))</td>
</tr>
<tr>
<td>((3)4)</td>
<td>((3)x)</td>
<td>((x)y)</td>
</tr>
<tr>
<td>((3)(4))</td>
<td>((3)(x))</td>
<td>((x)(y))</td>
</tr>
</tbody>
</table>

Now we will introduce the term *factors*, which we shall be using throughout the text.
**Factors**

The numbers or variables that are multiplied in a multiplication problem are called **factors**.

If \( a \cdot b = c \), then \( a \) and \( b \) are **factors** of \( c \).

For example, in \( 3 \cdot 5 = 15 \), the numbers 3 and 5 are factors of the product 15. In \( 2 \cdot 15 = 30 \), the numbers 2 and 15 are factors of the product 30. Note that 30 has many other factors. Since \( 5 \cdot 6 = 30 \), the numbers 5 and 6 are also factors of 30. Since \( 3x \) means 3 times \( x \), both the 3 and the \( x \) are factors of \( 3x \).

### 2 Simplify Fractions

Now we have the necessary information to discuss **fractions**. The top number of a fraction is called the **numerator**, and the bottom number is called the **denominator**. In the fraction \( \frac{3}{5} \), the 3 is the **numerator** and the 5 is the **denominator**.

#### Helpful Hint

Consider the fraction \( \frac{3}{5} \). There are equivalent methods of expressing this fraction, as illustrated below.

\[
\frac{3}{5} = \frac{3}{5} = \frac{3}{5} = \frac{3}{5}
\]

In general, \( \frac{a}{b} = \frac{a}{b} = \frac{a}{b} = \frac{a}{b} \)

#### Understanding Algebra

**GCF** stands for “greatest common factor.” It is the largest number that divides evenly into the two given numbers. The GCF of 12 and 18 is 6.

A fraction is **simplified**, or **reduced to its lowest terms**, when the numerator and denominator have no common factors other than 1. To simplify a fraction, follow these steps.

#### To Simplify a Fraction

1. Find the largest number that will divide (without remainder) both the numerator and the denominator. This number is called the **greatest common factor** (GCF).
2. Then divide both the numerator and the denominator by the greatest common factor.
Simplify each fraction. If a fraction is already simplified, so state.

11. \[ \frac{10}{15} \]
12. \[ \frac{40}{10} = \frac{4}{1} = \frac{4}{1} \]
13. \[ \frac{6}{24} \]

\[ \text{GCF} = 10 \]

\[ \frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3} \]

5 = Greatest Common Factor

14. \[ \frac{19}{25} \]

15. \[ \frac{36}{76} \]

16. \[ \frac{16}{72} \]

\[ \text{Reduced} \]

\[ \frac{16}{72} = \frac{8}{36} = \frac{4}{18} = \frac{2}{9} \]
3 Multiply Fractions

To multiply two or more fractions, multiply their numerators together and multiply their denominators together.

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}
\]

EXAMPLE 2 Multiply \(\frac{3}{13}\) by \(\frac{5}{11}\).

Solution

\[
\frac{3}{13} \cdot \frac{5}{11} = \frac{3 \cdot 5}{13 \cdot 11} = \frac{15}{143}
\]

Now Try Exercise 39

Before multiplying fractions, to help avoid having to simplify an answer, we often divide both a numerator and a denominator by a common factor.
Find each product or quotient. Simplify the answer.

39. \( \frac{1}{3} \times \frac{4}{5} \)

40. \( \frac{6}{13} \times \frac{7}{17} \)

41. \( \frac{5}{12} \times \frac{4}{15} \)

42. \( \frac{36}{48} \times \frac{16}{45} \)
4 Divide Fractions

To divide one fraction by another, invert the divisor (the second fraction if written with ÷) and proceed as in multiplication.

To Divide Fractions

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}
\]

Sometimes, rather than being asked to add, subtract, multiply, or divide, you may be asked to evaluate an expression. To evaluate an expression means to obtain the answer to the problem using the operations given.

EXAMPLE 5 Evaluate a) \(\frac{3}{5} \div \frac{5}{6}\) b) \(\frac{3}{8} \div 12\).

Solution

a) \(\frac{3}{5} \div \frac{5}{6} = \frac{3}{5} \cdot \frac{6}{5} = \frac{3 \cdot 6}{5 \cdot 5} = \frac{18}{25}\)

b) Write 12 as \(\frac{12}{1}\). Then invert the divisor and multiply.

\[
\frac{3}{8} \div 12 = \frac{3}{8} \div \frac{12}{1} = \frac{1}{8} \cdot \frac{1}{\frac{12}{4}} = \frac{1}{32}
\]

Now Try Exercise 47
50. \( \frac{5}{12} \div \frac{4}{3} \)

\[
\frac{5}{12} \div \frac{4}{3} = \frac{5}{12} \times \frac{3}{4} = \frac{5}{16}
\]

54. \( \frac{4}{5} \div \frac{8}{15} \)

In \( \times \) or \( \div \)

You must change
All Mixed #s
To Improper Fractions!

\[
4 \frac{4}{5} = \frac{24}{5}
\]

\[
\frac{24}{5} \div \frac{8}{3} = \frac{3}{1} \times \frac{3}{12}
\]

\[
= \frac{9}{1} = 9
\]
Add and Subtract Fractions

Fractions that have the same (or a common) denominator can be added or subtracted. To add or subtract fractions with the same denominator, add or subtract the numerators and keep the common denominator.

To Add and Subtract Fractions

\[
\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c} \quad \text{or} \quad \frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}
\]

EXAMPLE 6 a) Add \( \frac{6}{15} + \frac{2}{15} \).

Solution

\[
\frac{6}{15} + \frac{2}{15} = \frac{6 + 2}{15} = \frac{8}{15}
\]

b) Subtract \( \frac{8}{13} - \frac{5}{13} \).

\[
\frac{8}{13} - \frac{5}{13} = \frac{8 - 5}{13} = \frac{3}{13}
\]

Now Try Exercise 55

To add (or subtract) fractions with unlike denominators, we must first rewrite each fraction with a common denominator. The smallest number that is divisible by two or more denominators is called the least common denominator or LCD. If you have forgotten how to find the least common denominator, review Appendix B now.

EXAMPLE 7 Add \( \frac{1}{2} + \frac{1}{5} \).

Solution We cannot add these fractions until we rewrite them with a common denominator. Since the lowest number that both 2 and 5 evenly divide into is 10, we will first rewrite both fractions with the least common denominator of 10.

\[
\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} \quad \text{and} \quad \frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}
\]

Now we add.

\[
\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}
\]
Add or subtract. Simplify each answer.

55. \( \frac{3}{8} + \frac{2}{8} \)
56. \( \frac{18}{36} - \frac{5}{36} \)
57. \( \frac{3}{14} - \frac{1}{14} \)
58. \( \frac{1}{4} + \frac{3}{4} \)
59. \( \frac{4}{5} + \frac{6}{15} \)
60. \( \frac{7}{8} - \frac{5}{6} \)
61. \( \frac{9}{17} + \frac{2}{34} \)
62. \( \frac{3}{7} + \frac{17}{35} \)

\[
\frac{3}{8} + \frac{2}{8} = \frac{5}{8}
\]
\[
\frac{18}{36} - \frac{5}{36} = \frac{13}{36}
\]
\[
\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1
\]

\[
\frac{7}{5} \cdot \frac{3}{5} = \frac{21}{24} = \frac{21}{24}
\]
\[
\frac{8}{16} \quad \text{(24 = LCD)}
\]

\[
\frac{5 \cdot \frac{4}{9}}{6 \cdot \frac{4}{9}}\]

**Less Common Denominator**
6 Change Mixed Numbers to Fractions and Vice Versa

Consider the number $5 \frac{2}{3}$. This is an example of a mixed number. A mixed number consists of a whole number followed by a fraction. The mixed number $5 \frac{2}{3}$ means $5 + \frac{2}{3}$. We can change $5 \frac{2}{3}$ to a fraction as follows.

$$5 \frac{2}{3} = 5 + \frac{2}{3} = \frac{15}{3} + \frac{2}{3} = \frac{17}{3}$$

Notice that we expressed the whole number, 5, as a fraction with a denominator of 3, then added the fractions.

EXAMPLE 9 Change $7 \frac{3}{8}$ to a fraction.

Solution

$$7 \frac{3}{8} = 7 + \frac{3}{8} = \frac{56}{8} + \frac{3}{8} = \frac{59}{8}$$

Now Try Exercise 25

Now consider the fraction $\frac{17}{3}$. We convert it to a mixed number as follows.

$$\frac{17}{3} = \frac{15}{3} + \frac{2}{3} = 5 + \frac{2}{3} = 5 \frac{2}{3}$$

Notice we wrote $\frac{17}{3}$ as a sum of two fractions, each with the denominator of 3. The first fraction being added, $\frac{15}{3}$, is the equivalent of the largest integer that is less than $\frac{17}{3}$. 
Convert each mixed number to a fraction.

23. $2 \frac{13}{15}$  
24. $15 \frac{1}{3}$  
25. $7 \frac{2}{3}$  
26. $14 \frac{3}{4}$  
27. $3 \frac{5}{18}$  
28. $2 \frac{2}{9}$  
29. $9 \frac{6}{17}$  
30. $3 \frac{3}{32}$

Write each fraction as a mixed number.

31. $\frac{7}{4}$  
32. $\frac{18}{7}$  
33. $\frac{13}{4}$  
34. $\frac{9}{2}$  
35. $\frac{32}{7}$  
36. $\frac{110}{20}$  
37. $\frac{86}{14}$  
38. $\frac{72}{14}$

$14 \frac{3}{9} = (\frac{57}{4})$  
$4 \times 14 = 56$  
$\frac{56}{3} = \frac{59}{3}$

$2\frac{2}{9} = \frac{20}{9}$  
$2 \times 9 = 18$  
$+ \frac{2}{20}$

$1\frac{8}{7} = 2\frac{4}{7}$  
$\frac{24}{7} = \frac{24}{7}$

$7 \frac{118}{17}$
86. **Baking Turkey** The instructions on a turkey indicate that a 12- to 16-pound turkey should bake at 325°F for about 22 minutes per pound. Josephine Nickola is planning to bake a 13\(\frac{1}{2}\)-pound turkey. Approximately how long should the turkey be baked?
88. **Pants Inseam** The inseam on a new pair of pants is 32 inches.

If Don O'Neal's inseam is $29\frac{3}{8}$ inches, by how much will the pants need to be shortened?
98. **Soda** If five 2-liter bottles of soda are split evenly among 30 people, how many liters of soda will each person get?
\[ 1 = \frac{14}{14} \]
\[ \frac{5}{6} \cdot \frac{2}{2} = \frac{5}{14} \text{ Adv} \]
\[ -2 \frac{3}{14} = \frac{9}{14} \]
\[ \boxed{3 \frac{11}{14}} \]