1. \[5 - 2(3x - 5y + 2) - (x - y + 2) - 3(-2x - 3y + 4) = 6\]
   \[-6x + 10y - 4 - x + y - 2 + 6x + 9y = 12 - 6\]
   \[-19 - 1x + 20y\]

2. \[-8 = x - 14 + 14\]
   \[-8 = x\]
   \[
   \frac{3}{3} = \frac{2}{2}
   \]
   \[
   x = \frac{7}{7}
   \]

3. \[-3x = \frac{-21}{-3}\]
   \[x = 7\]

4. \[
\frac{2}{3} \cdot \frac{5}{2} = \frac{5}{3}\]

5. \[
\frac{3}{4}x = \frac{5}{6}\]
   \[
   x = \frac{10}{9}\]

6. \[
\frac{12}{1} \left(\frac{3}{4}x = \frac{-5}{6}\right) \frac{12}{1}\]
   \[
3(3x) = (5)\]
   \[
9x = \frac{-10}{9}\]
   \[
   x = \frac{-10}{9}\]

7. \[
-\frac{1}{9} \left(-8x = \frac{16}{9}\right) \frac{-1}{9}\]
   \[
   x = \frac{-2}{9}\]

8. \[
-2x - 8 = 12 + 8\]
   \[
   \frac{-2x = 20}{-2} \frac{10}{-2}\]
   \[
   x = -10\]

9. \[
6x - 7 = -5x + 3\]
   \[
   +5x + 5x\]
   \[
   11x - 7 = 3\]
   \[
   +7 + 7\]
   \[
   Constant s to Right\]
   \[
\frac{11x = 10}{11}\]
   \[
   x = \frac{10}{11}\]
\[ \frac{89}{1} \left( \frac{1}{2} x + \frac{5}{1} \right) = \frac{1}{8} \]

\[ 4(x) + 8(5) = 4(1) \]

\[ 4x + 40 = 1 \]

\[ 4x = -39 \]

\[ x = -\frac{39}{4} \]
2.5 Solving Linear Equations with the Variable on Both Sides of the Equation

1. Solve Equations with the Variable on Both Sides of the Equal Sign

The equation $4x + 6 = 2x + 4$ contains the variable $x$, on both sides of the equal sign. To solve equations of this type, we must use the appropriate properties to rewrite the equation with all terms containing the variable on only one side of the equal sign and all terms not containing the variable on the other side of the equal sign. Following is a general procedure, similar to the one outlined in Section 2.4, that can be used to solve linear equations with the variable on both sides of the equal sign. The steps in the procedure are only guidelines to use. For example, there may be times when you may choose to use the distributive property, step 2, before multiplying both sides of the equation by the LCD, step 1. We will illustrate this in Examples 8 and 9.

To Solve Linear Equations with the Variable on Both Sides of the Equal Sign

1. If the equation contains fractions, multiply both sides of the equation by the least common denominator. This will eliminate fractions from the equation.
2. Use the distributive property to remove parentheses.
3. Combine like terms on the same side of the equal sign.
4. Use the addition property to rewrite the equation with all terms containing the variable on one side of the equal sign and all terms not containing the variable on the other side of the equal sign. It may be necessary to use the addition property twice to accomplish this goal. You will eventually get an equation of the form $ax = b$.
5. Use the multiplication property to isolate the variable. This will give a solution of the form $x = \text{some number}$.
6. Check the solution in the original equation.
The steps listed on page 129 are basically the same as the steps listed in the boxed procedure on page 120, except that in step 4 you may need to use the addition property more than once to obtain an equation of the form \( ax = b \).

Remember that our goal in solving an equation is to isolate the variable, that is, to get the variable alone on one side of the equation.

Consider the equation \( 3x + 4 = x + 12 \) which contains no fractions or parentheses, and no like terms on the same side of the equal sign. Therefore, we start with step 4, the addition property. We will use the addition property twice in order to obtain an equation where the variable appears on only one side of the equal sign. We begin by subtracting \( x \) from both sides of the equation to get all the terms containing the variable on the left side of the equation. This will give the following:

\[
\begin{align*}
3x + 4 &= x + 12 \\
3x - x + 4 &= x - x + 12 \\
or &
2x + 4 &= 12
\end{align*}
\]

Addition property

Variable appears only on left side of equal sign.

Notice that the variable, \( x \), now appears on only one side of the equation. However, \( +4 \) still appears on the same side of the equal sign as the \( 2x \). We use the addition property a second time to get the term containing the variable by itself on one side of the equation. Subtracting 4 from both sides of the equation gives \( 2x = 8 \), which is an equation of the form \( ax = b \).

\[
\begin{align*}
2x + 4 &= 12 \\
2x + 4 - 4 &= 12 - 4 \\
2x &= 8
\end{align*}
\]

Addition property

\( x \)-term is now isolated.

The \( x \)-term, \( 2x \), is now by itself on one side of the equation. Therefore, we have isolated the \( x \)-term on the left side of the equation. We can now use the multiplication property, step 5, to isolate the variable and solve the equation for \( x \). We divide both sides of the equation by 2 to isolate the variable and solve the equation.

\[
\begin{align*}
2x &= 8 \\
\frac{2x}{2} &= \frac{8}{2} \\
x &= 4
\end{align*}
\]

Multiplication property

\( x \) is now isolated.

The solution to the equation is 4.
3 Identify Identities and Contradictions

Thus far all the equations we have solved have had a single value for a solution. Equations of this type are called **conditional equations**, for they are only true under specific conditions. Some equations, as in Example 11, are true for infinitely many values of \( x \). Equations that are true for infinitely many values of \( x \) are called **identities**. A third type of equation, as in Example 12, has no solution and is called a **contradiction**.

**EXAMPLE 11** Solve the equation \( 5x - 5 - 2x = 3(x - 2) + 1 \).

**Solution**

\[
\begin{align*}
5x - 5 - 2x &= 3(x - 2) + 1 \\
3x - 5 &= 3x - 6 + 1 \\
3x - 5 &= 3x - 5 \\
0 &= 0
\end{align*}
\]

Distributive property was used.

Like terms were combined.

Since the same expression appears on both sides of the equal sign, the statement is true for infinitely many values of \( x \). If we continue to solve this equation further, we might obtain

\[
\begin{align*}
3x &= 3x \\
0 &= 0
\end{align*}
\]

\( 3 \) was added to both sides.

\( 0 \) was subtracted from both sides.

**NOTE:** The solution process could have been stopped at \( 3x - 5 = 3x - 5 \). Since one side is identical to the other side, the equation is true for infinitely many values of \( x \). The solution to this equation is **all real numbers**. When solving an equation like the equation in Example 11, that is always true, write your answer as **“all real numbers.”**

† Now Try Exercise 47

**EXAMPLE 12** Solve the equation \( -2x + 5 + 3x = 5x - 4x + 7 \).

**Solution**

\[
\begin{align*}
-2x + 5 + 3x &= 5x - 4x + 7 \\
x + 5 &= x + 7 \\
x - x + 5 &= x - x + 7 \\
5 &= 7
\end{align*}
\]

Like terms were combined.

Subtract \( x \) from both sides.

False

**NOTE:** When solving an equation, if you obtain an obviously false statement, as in this example, the equation has **no solution**. No value of \( x \) will make the equation a true statement. When solving an equation like the equation in Example 12, that is never true, write your answer as **“no solution.”** An answer left blank may be marked wrong.
20. \(5x + 7 = 3x + 5\)

Move Variables to Left

\[
\begin{align*}
5x + 7 &= 3x + 5 \\
-3x &
\end{align*}
\]

\[
\begin{align*}
2x + 7 &= 5 \\
-7 &
\end{align*}
\]

\[
\begin{align*}
\frac{2x}{2} &= \frac{-2}{2} \\
x &= -1
\end{align*}
\]

30. \(3x - 5 + 9x = 2 + 4x + 9\)

Move Constants to Right

\[
\begin{align*}
3x + 7 &= 3x + 5 \\
-7 &
\end{align*}
\]

\[
\begin{align*}
2x &= -2x + 5 \\
-5 &
\end{align*}
\]

\[
\begin{align*}
\frac{2x}{2} &= \frac{-2x}{2} \\
-1 &= x
\end{align*}
\]

36. \(4(x - 3) + 2 = 2x + 8\)

\[
\begin{align*}
4(x - 7) + 2 &= 2x + 8 \\
4x - 12 + 2 &= 2x + 8 \\
4x - 10 &= 2x + 8 \\
-2x &
\end{align*}
\]

\[
\begin{align*}
2x - 10 &= 8 \\
+10 &+10
\end{align*}
\]

\[
\begin{align*}
\frac{2x}{2} &= \frac{18}{2} \\
x &= 9
\end{align*}
\]

26. \(8.71 - 2.44x = 11.02 - 5.74x\)

66. \(-2(-3x + 5) + 6 = 4(x - 2)\)

44. \(\frac{3}{4}x + \frac{1}{2} = \frac{1}{2}x\)

\[
\begin{align*}
\frac{3}{4}x + \frac{1}{2} &= \frac{1}{2}x \\
1(3x) + 2(1) &= 1(x) \\
3x + 2 &= 2x \\
-2x &
\end{align*}
\]

\[
\begin{align*}
x + 2 &= 0 \\
-2 &-2
\end{align*}
\]

\[
\begin{align*}
x &= -2
\end{align*}
\]
76. \[ \frac{5}{12}(x + 2) = \frac{2}{3}(2x + 1) + \frac{1}{6} \]

Distribute 1st

\[ \frac{5}{12}(x + 2) = \frac{2}{3}(2x+1) + \frac{1}{6} \]

\[ \frac{12}{1} \left[ \frac{5}{12}x + \frac{5}{6} \right] = \frac{4}{3}x + \frac{2}{3} + \frac{1}{6} \]

Clear Fraction

\[ \text{LCD}=12 \]

\[ 1(5x) + 2(5) = 4(4x) + 4(2) + 1(12) \]

\[ 5x + 10 = 16x + 8 + 2 \]

\[ 5x + 10 = 16x + 10 \]

\[ -5x \quad -5x \]

\[ 10 = 11x + 10 \]

\[ -10 \quad -10 \]

48. \[ 3(y - 1) + 9 = 8y + 6 - 5y \]

Combine Like Terms

Same Side of Equation

\[ 3y - 3 + 9 = 8y + 6 - 5y \]

\[ 3y + 6 = 3y + 6 \]

\[ -3y \quad -3y \]

\[ 6 = 6 \]

True

Identity

All Reals
\[
\left[ \frac{1}{2} (2d+4) = \frac{1}{3} (4d-4) \right] \text{ Distribute}
\]

\[
3(y-1) + 9 = 8y + 6 - 5y
\]

\[
\frac{24}{25}
\]

\[
\frac{h}{1} \left[ \frac{5(x-4)}{4} = \frac{5(2x-3)}{3} \right] \cdot \frac{h}{1}
\]
10. \( 7x + 3 = 2x - 1 \)
\[
\begin{align*}
7x - 2x &= -1 - 3 \\
5x &= -4 \\
x &= \frac{-4}{5} \\
x &= -\frac{4}{5}
\end{align*}
\]
12. \( 5(x - 2) + 6 = -9x - 3 \)
\[
\begin{align*}
5x - 10 + 6 &= -9x - 3 \\
5x - 9 &= -9x - 3 \\
+4x &= +4x \\
9x - 4 &= -3 \\
+4 &= +4 \\
9x^2 + 4 &= 4x^4
\end{align*}
\]
13. \( 2(x + 1) = -3(x - 7) - 1 \)
\[
\begin{align*}
2x - 4 &= -3x + 1\frac{1}{2} \\
-3x &= -3\frac{1}{2} \\
x &= \frac{3}{2} \\
x &= \frac{3}{2}
\end{align*}
\]
15. \( \frac{2}{3} x - \frac{1}{2} = \frac{1}{3} \)
\[
\begin{align*}
2x - 3 &= -2\frac{1}{2} \\
2x &= -2\frac{1}{2} + 3 \\
x &= -\frac{2}{2} + 1 \\
x &= 1
\end{align*}
\]
16. \( 0.01x + 8 = 0.005(x - 2) \)
\[
\begin{align*}
x + 800 &= 3x - 6 \\
-800 &= 2x - 6 \\
+6 &= +6 \\
x &= 816
\end{align*}
\]
18. \( 7(x - 3) = x + 6x - 2 \)
\[
\begin{align*}
7x - 21 &= 7x - 2 \\
-7x &= -7x \\
-21 &= -21
\end{align*}
\]
False
Contradiction
No Solution