In Exercises 41–44, find the missing quantity. Use the ideal gas law, \( P = \frac{KT}{V} \), where \( P \) is pressure, \( T \) is temperature, \( V \) is volume, and \( K \) is a constant.

43. \( T = 30, P = 3, K = 0.5 \)

\[
P = \frac{KT}{V}
\]

\[
\frac{V}{1} = \frac{3}{1} = \frac{(0.5)(30)}{V}
\]

\[
\frac{3V}{3} = \frac{15}{3}
\]

\[
V = 5
\]
55. $3 - 2r = n$, for $r$

\[\begin{align*}
\frac{3 - 2r}{-3} &= \frac{n}{-3} \\
\frac{-2r}{-3} &= \frac{n - 3}{-3} \\
r &= \frac{n - 3}{-2} \quad \text{or} \quad r = \frac{n - 3}{2}
\end{align*}\]
65. \[ A = \frac{m + d}{2}, \text{ for } m \]

\[
\frac{2A}{1} = \frac{m + d}{2}
\]

\[
2A = m + d
\]

\[
-d -d
\]

\[
2A - d = m
\]
79. \( y + 3 = -\frac{1}{3}(x - 4) \)

\[
\begin{align*}
    y + 3 &= -\frac{1}{3}(x - 4) \\
    -3 &= -\frac{1}{3}x + \frac{4}{3} \\
    -3 \quad &\underline{+ 3} \\
    y &= -\frac{1}{3}x - \frac{5}{3}
\end{align*}
\]
76. \(-12 = -2x - 3y\)

\[
\begin{align*}
2x & - 12 = -2x - 3y \\
2x & - 12 = -3y \\
-3 & = -3y \\
-y & = \frac{2x - 12}{3} \\
y & = \frac{2x - 12}{-3} \\
y & = \frac{2x}{-3} - \frac{12}{-3} \\
y & = \frac{2}{3}x + 4
\end{align*}
\]
2.8 Inequalities in One Variable

1 Solve linear inequalities.
2 Solve linear inequalities that have all real numbers as their solution, or have no solution.

1 Solve Linear Inequalities

The is-greater-than symbol, $>$, and is-less-than symbol, $<$, were introduced in Section 1.5. The symbol $\geq$ means is greater than or equal to and $\leq$ means is less than or equal to. A mathematical statement containing one or more of these symbols is called an inequality. The direction of the symbol is sometimes called the sense or order of the inequality.

Examples of Inequalities in One Variable

$$x + 3 < 5 \quad x + 4 \geq 2x - 6 \quad 4 > -x + 3$$

To solve an inequality, we must get the variable by itself on one side of the inequality symbol. To do this, we make use of properties very similar to those used to solve equations. Here are four properties used to solve inequalities. Later in this section, we will introduce two additional properties.

<table>
<thead>
<tr>
<th>Properties Used to Solve Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>For real numbers, $a$, $b$, and $c$ :</td>
</tr>
<tr>
<td>1. If $a &gt; b$, then $a + c &gt; b + c$.</td>
</tr>
<tr>
<td>2. If $a &gt; b$, then $a - c &gt; b - c$.</td>
</tr>
<tr>
<td>3. If $a &gt; b$ and $c &gt; 0$, then $ac &gt; bc$.</td>
</tr>
<tr>
<td>4. If $a &gt; b$ and $c &gt; 0$, then $\frac{a}{c} &gt; \frac{b}{c}$.</td>
</tr>
</tbody>
</table>

$$3 > 2$$
$$3 \cdot 4 > 2 \cdot 4$$
$$-12 < -8$$

$$3 > 2$$
$$3 \cdot 5 > 2 \cdot 5$$
$$15 > 10$$

$$\frac{-5x}{-5} < \frac{25}{-5}$$

$$x > -5$$
14. $x - 3 \geq -9$

\[
x - 3 \geq -9 \\
+3 \quad +3 \\
x \geq -6
\]

38. $-2(w + 3) \leq 4w + 5$

\[
-2(w + 3) \leq 4w + 5 \\
-2w - 6 \leq 4w + 5 \\
-4w \quad -4w \\
-6w - 6 \leq 5 \\
+6 \\
-6w \leq 11 \\
\frac{-6w}{-6} \\
w \geq -\frac{11}{6}
\]

52. \(\frac{x}{7} - 1 \geq \frac{x}{8}\)

\[
\frac{56}{7} \left[\frac{x}{7} - 1 \geq \frac{x}{8}\right] \\
8x - 1(56) \geq 7x \\
8x - 56 \geq 7x \\
-8x \\
-8x
\]

Reverse the sign if you divide by a negative.

\[
x \geq 56
\]

22. $6n - 12 < -12$

\[
6n < 0 \\
\frac{6n}{6} \\
0 < n
\]

56. $56 < x$

\[
\frac{-56}{-1} \quad \frac{56}{-1} \\
-56 \geq -x \\
-2w - 6 \leq 4w + 5 \\
+2w \\
-6 \geq 6w + 5 \\
-5 \\
-11 \leq 6w \\
\frac{-11}{6} \leq w
\]

Put \(w\) on the left side.

\[
w \geq -\frac{11}{6}
\]
Additional Properties Used to Solve Inequalities

5. If \( a > b \) and \( c < 0 \), then \( ac < bc \).

6. If \( a > b \) and \( c < 0 \), then \( \frac{a}{c} < \frac{b}{c} \).

18. \( 6 \leq -3 - x \)

20. \( -12 \geq -3b \)

\[
\begin{align*}
\frac{-12}{-3} & \geq \frac{-3b}{-3} \\
4 & \leq b \\
\text{Rewrite:} & \\
& b \geq 4
\end{align*}
\]
**Helpful Hint**

- $a > x$ means $x < a$  Note that both inequality symbols point to $x$.
- $a < x$ means $x > a$  Note that both inequality symbols point to $a$.

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3 &gt; x$ means $x &lt; -3$</td>
</tr>
<tr>
<td>$-5 \leq x$ means $x \geq -5$</td>
</tr>
</tbody>
</table>

34. $-4n - 6 > 4n - 20$

54. $\frac{3}{5}r - 9 < \frac{3}{8}r$
2. Solve Linear Inequalities That Have All Real Numbers as Their Solution, or Have No Solution

In Examples 8 and 9, we illustrate two special types of inequalities. Example 8 is an inequality that is true for all real numbers, and Example 9 is an inequality that is never true for any real number.

**Example 8** Solve the inequality \(2(x + 3) \leq 5x - 3x + 8\), and graph the solution on a number line.

**Solution**

\[
\begin{align*}
2(x + 3) & \leq 5x - 3x + 8 \\
2x + 6 & \leq 2x + 8 \\
2x - 2x + 6 & \leq 2x - 2x + 8 \\
6 & \leq 8
\end{align*}
\]

Since 6 is always less than or equal to 8, the solution is all real numbers (Fig. 2.13).

40. \(y + 4 \geq y - 3\)

44. \(-2(x + 3) + x = 3x + 3 - x\)

46. \(x < x + 2\)

47. \(x > x + 1\)

52. \(x \leq 5\)
(1) \[ 4x + 2x - 3 - 3y + 6 + 2y \]
\[ = (6x - 3y + 3) \]

(2) \[ -2 (3 - x) - (3x - 8) \]
\[ = -6 + 2x - 3x + 8 \]
\[ = -1x + 2 \]
\[ = -x + 2 \]
1. \[ x + 3 = 2 \]
\[ \frac{-3}{-3} \]
\[ x = -1 \]

2. \[ -14 = 3x + 5 \]
\[ \frac{-5}{-5} \]
\[ \frac{-19 = 3x}{3} \]
\[ x = \frac{-19}{3} \]

3. \[ -4x = 7 \]
\[ \frac{-4}{-4} \]
\[ x = -\frac{7}{4} \]

4. \[ -x = \frac{8}{-1} \]
\[ x = -8 \]
\[
\frac{28}{1} \left[ \frac{3}{4} x + \frac{6}{7} \right] = -\frac{3}{14} \left[ \frac{28}{1} \right]
\]

LCM = 28

\[
7(3x) + 4(6) = (-3)(2)
\]

\[
21x + 24 = -6
\]

\[
21x = -30
\]

\[
\frac{21x}{21} = \frac{-30}{21}
\]

\[
x = \frac{-30}{21} = \frac{-10}{7}
\]
\[ 2x + 1 = 2(x+1) \]
\[
\begin{align*}
2x + 1 &= 2x + 2 \\
-2x &= -2x \\
\hline
1 &= 2
\end{align*}
\]
False.

Contradiction.
No Solution.

\[ 3 - (x+6) + 4 = 5x + 2x \]
\[
\begin{align*}
3 - x - 6 + 4 &= 5x + 2x \\
-x + 1 &= 5x + 2x \\
+1x &\quad +1x
\end{align*}
\]
\[
\begin{align*}
1 &= 5x + 3x \\
-5 &= -5
\end{align*}
\]
\[
\begin{align*}
-\frac{4}{3} &= \frac{3x}{3} \\
\frac{-4}{3} &= x
\end{align*}
\]
\[
\begin{align*}
\frac{12}{15} \left[ \frac{3x}{4} + \frac{4(x-2)}{3} \right] &= \frac{3(2x+4)}{9} + \frac{x}{2}
\end{align*}
\]

\[
3(3x) + \frac{4(4)(x-2)}{16(x-2)} = 3(5)(2x+4) + 6x
\]

\[
9x + \frac{18x - 32}{24x} = 18x + 36 + 6x
\]

\[
\frac{25x - 32}{-24x} = \frac{24x + 36}{-24x}
\]

\[
X - 32 = 36
\]

\[
+32 + 32
\]

\[
X = 68
\]