3.1 Changing Application Problems into Equations

1. Translate phrases into mathematical expressions.
2. Express the relationship between two related quantities.
3. Write expressions involving multiplication.
4. Translate applications into equations.

## Helpful Hint

It is important that you prepare for this chapter carefully. Make sure you read the book and work the examples carefully. *Attend class every day, and most of all, work all the exercises assigned to you.*

As you read through the examples in the rest of the chapter, think about how they can be expanded to other, similar problems. For example, in Example 1a) we will state that the distance, \( d \), increased by 10 miles, can be represented by \( d + 10 \). You can generalize this to other, similar problems. For example, a weight, \( w \), increased by 15 pounds, can be represented as \( w + 15 \).

One practical advantage of knowing algebra is that you can use it to solve everyday problems involving mathematics. For algebra to be useful in solving everyday problems, you must first be able to translate application problems into mathematical language. One purpose of this section is to help you take an application problem, also referred to as a *word* or *verbal problem*, and write it as a mathematical equation.

Often the most difficult part of solving an application problem is translating it into an equation. Before you can translate a problem into an equation, you must understand the meaning of certain words and phrases and how they are expressed mathematically. Table 3.1 is a list of selected words and phrases and the operations they imply. We used the variable \( x \). However, any variable could have been used.
### Table 3.1

<table>
<thead>
<tr>
<th>Word or Phrase</th>
<th>Operation</th>
<th>Statement</th>
<th>Algebraic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Added to</td>
<td>Addition</td>
<td>8 added to a number</td>
<td>x + 8</td>
</tr>
<tr>
<td>More than</td>
<td></td>
<td>6 more than a number</td>
<td>x + 6</td>
</tr>
<tr>
<td>Increased by</td>
<td></td>
<td>A number increased by 3</td>
<td>x + 3</td>
</tr>
<tr>
<td>The sum of</td>
<td></td>
<td>The sum of a number and 4</td>
<td>x + 4</td>
</tr>
<tr>
<td>Subtracted from</td>
<td>Subtraction</td>
<td>6 subtracted from a number</td>
<td>x - 6</td>
</tr>
<tr>
<td>Less than</td>
<td></td>
<td>2 less than a number</td>
<td>x - 2</td>
</tr>
<tr>
<td>Decreased by</td>
<td></td>
<td>A number decreased by 5</td>
<td>x - 5</td>
</tr>
<tr>
<td>The difference between</td>
<td></td>
<td>The difference between a number and 9</td>
<td>x - 9</td>
</tr>
<tr>
<td>Multiplied by</td>
<td>Multiplication</td>
<td>A number multiplied by 6</td>
<td>6x</td>
</tr>
<tr>
<td>The product of</td>
<td></td>
<td>The product of 4 and a number</td>
<td>4x</td>
</tr>
<tr>
<td>Twice a number</td>
<td></td>
<td>Twice a number</td>
<td>2x</td>
</tr>
<tr>
<td>3 times a number, etc.</td>
<td></td>
<td>20% of a number</td>
<td>0.20x</td>
</tr>
<tr>
<td>Of, when used with a percent or fraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Divided by</td>
<td>Division</td>
<td>A number divided by 8</td>
<td>( \frac{x}{8} )</td>
</tr>
<tr>
<td>The quotient of</td>
<td></td>
<td>The quotient of a number and 6</td>
<td>( \frac{x}{6} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
<th>Algebraic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four more than twice a number</td>
<td>( 2x + 4 )</td>
</tr>
<tr>
<td>Two times less than 3 times a number</td>
<td>( 3x - 5 )</td>
</tr>
<tr>
<td>Three times the sum of a number and 8</td>
<td>( 3(x + 8) )</td>
</tr>
<tr>
<td>Twice the difference between a number and 4</td>
<td>( 2(x - 4) )</td>
</tr>
</tbody>
</table>

### Algebraic Statements

- \( 2x + 3 \)
  - Three more than twice a number
  - The sum of twice a number and 3
  - Twice a number increased by 3
  - Three added to twice a number

- \( 3x - 4 \)
  - Four less than 3 times a number
  - Three times a number, decreased by 4
  - The difference between 3 times a number and 4
  - Four subtracted from 3 times a number
In Exercises 11–30, express the statement as an algebraic expression. See Example 1.

11. The height, $h$, increased by 4 inches
12. The weight, $w$, increased by 20 pounds
13. The age, $a$, decreased by 5 years
14. The time, $t$, decreased by 3 hours
15. Five times the height, $h$
16. Seven times the length, $l$
17. Twice the distance, $d$
18. Three times the rate, $r$
19. One-half the age, $a$
20. One-third the weight, $w$

21. Five subtracted from $r$
22. Nine subtracted from $p$
23. $m$ subtracted from 8
24. $n$ subtracted from 4
25. Eight pounds more than twice the weight, $w$
26. Six inches more than 3 times the height, $h$
27. Four years less than 5 times the age, $a$
28. One mile more than $\frac{1}{2}$ the distance, $d$
29. One-third the weight, $w$, decreased by 7 pounds
30. One-fifth the height, $h$, increased by 2 feet
Express the Relationship between Two Related Quantities

Sometimes in a problem, two numbers are related to each other in a certain way. We often represent the simplest, or most basic, number that needs to be expressed as a variable, and the other as an expression containing that variable. Some examples follow.

<table>
<thead>
<tr>
<th>Statement</th>
<th>One Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two numbers differ by 5</td>
<td>$x$</td>
<td>$x + 5$</td>
</tr>
<tr>
<td>Mike’s age now and Mike’s age in 8 years</td>
<td>$x$</td>
<td>$x + 8$</td>
</tr>
<tr>
<td>One number is 6 times the other number</td>
<td>$x$</td>
<td>$6x$</td>
</tr>
<tr>
<td>One number is 12% less than the other</td>
<td>$x$</td>
<td>$0.12x$</td>
</tr>
</tbody>
</table>

34. Wilma ran 4 miles per hour faster than Natasha.
35. The United States won 3 times the number of medals that Finland won.
36. The distance to Georgia is $\frac{1}{2}$ the distance to Tennessee.
37. The Cadillac costs $200$ more than twice the cost of the Chevy.
38. Noah received 25 more votes than 3 times the number of votes that Tawnya received.
39. Jane’s grade was 2 points less than twice Teri’s grade.
40. Alberto’s salary was $2000$ greater than 4 times Nick’s salary.
41. $60$ divided between Kristen and Yvonne.
42. Drawka has 25 marbles. They are either red or blue marbles.
43. Together Don and Angela weigh 270 pounds.
44. Together Oliver and Dalene have 1053 clients.
3. **Write Expressions Involving Multiplication**

Consider the statement “the cost of 3 items at $5 each.” How would you represent this quantity using mathematical symbols? You would probably reason that the cost would be 3 times $5 and write $3 \cdot 5$ or $3(5)$.

Now consider the statement “the cost of $x$ items at $5 each.” How would you represent this statement using mathematical symbols? If you use the same reasoning, you might write $x \cdot 5$ or $x(5)$. Another way to write this product is $5x$. Thus, the cost of $x$ items at $5$ each could be represented as $5x$.

72. **Weight** Jason Mahar’s weight is $w$ pounds. Write an expression that represents his weight in ounces.

73. **Money** Susan Grady has $d$ dollars in her purse. Write an expression that represents this quantity of money in cents.

76. **Soil Delivery** Mary Vachon had top soil delivered to her house. The total cost included a delivery charge of $48 plus $60 per cubic yard of soil. Write an expression for the total cost if Mary has $x$ cubic yards of soil delivered.
88. Profits A company's profits in 2005 were $100 less than the company's profits in 2006. Write an expression for the difference in the 2006 and 2005 profits.

\[ x \quad x + 100 \]
Expressions Involving Percent

Example 7 involves percent. Before we leave this section, let's discuss expressions that involve percent further. Since percents are used so often, you must have a clear understanding of how to write expressions involving percent. Whenever we perform a calculation involving percent, we change the percent to a decimal number or a fraction first.

When shopping we may see a "25% off" sign. We assume that this means 25% of the original cost, even though this is not stated. If we let \( c \) represent the original cost, then 25% of the original cost would be represented as \( 0.25c \). Twenty-five percent off the original cost means the original cost, \( c \), decreased by 25% of the original cost. Twenty five percent off the original cost would be represented as \( c - 0.25c \).

![Diagram showing the relationship between original cost, decreased cost, and 25% of the original cost.]

78. **Salary Increase**  Charles Idon, an engineer, had a salary increase of 15% over last year's salary. Write an expression for this year's salary.

79. **Electricity Use**  Jean Olson's electricity use in 2006 decreased by 12% from her 2005 electricity use. Write an expression for her 2006 electricity use.

80. **Shirt Sale**  At a 25% off everything sale, Bill Winchief purchased a new shirt. If \( c \) represents the original cost, write an expression for the sale price of the shirt.

81. **Car Cost**  The cost of a new car purchased in Collier County included a 7% sales tax. If \( c \) represents the cost of the car before tax, write an expression for the total cost, including the sales tax.

\[
\begin{align*}
S + .15s &= 1.15s \\
1.00 &= .15s \\
S &= .88c \\
&= .88c \\
C &- .25c = .75c \\
c + .07c &= 1.07c
\end{align*}
\]
4 Translate Applications into Equations

Now we will explain how to write application problems as equations. The word *is* in an application problem often means *is equal to* and is represented by an equal sign. Some examples of statements written as equations follow.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Six times a number <em>is</em> 42.</td>
<td>$6x = 42$</td>
</tr>
<tr>
<td>Five more than twice a number <em>is</em> 4.</td>
<td>$2x + 5 = 4$</td>
</tr>
<tr>
<td>A number decreased by 4 <em>is</em> 3 more than twice the number.</td>
<td>$x - 4 = 2x + 3$</td>
</tr>
<tr>
<td>The sum of a number and the number increased by 4 <em>is</em> 60.</td>
<td>$x + (x + 4) = 60$</td>
</tr>
<tr>
<td>Twice the difference of a number and 3 <em>is</em> the sum of the number and 20.</td>
<td>$2(x - 3) = x + 20$</td>
</tr>
<tr>
<td>A number increased by 15% <em>is</em> 120.</td>
<td>$x + 0.15x = 120$</td>
</tr>
<tr>
<td>Six less than three times a number <em>is</em> one-fourth the number.</td>
<td>$3x - 6 = \frac{1}{4}x$</td>
</tr>
</tbody>
</table>

Now let's translate some equations into statements. Some examples of equations written as statements follow. We will write only two statements for each equation, but remember there are other ways these equations can be written.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x - 4 = 4x + 3$</td>
<td>Four less than 3 times a number <em>is</em> 3 more than 4 times the number.</td>
</tr>
<tr>
<td></td>
<td>Three times a number, decreased by 4 <em>is</em> 4 times the number, increased by 3.</td>
</tr>
<tr>
<td></td>
<td>Three times the difference between a number and 2 <em>is</em> 4 less than 6 times the number.</td>
</tr>
<tr>
<td>$3(x - 2) = 6x - 4$</td>
<td>The product of 3 and the difference between a number and 2 <em>is</em> 6 times the number, decreased by 4.</td>
</tr>
</tbody>
</table>
106. Numbers One-third of the sum of a number and 12 is 5.

107. Even Integers For two consecutive even integers, the sum of the smaller and twice the larger is 22.

108. Odd Integers For two consecutive odd integers, the sum of 3 times the smaller and the larger is 14.

109. Earnings John Jones earns $12.50 per hour. If he works $h$ hours his earnings will be $150.

110. Employees The T. W. Wilson company plans to increase its number of employees by 20 per year. The increase in the number of employees in $t$ years will be 120.

111. Top Soil Abe Mantell purchased $x$ bags of top soil at a cost of $2.99 a bag. The total he paid was $17.94.

112. Plants Mark Ernsthausen purchased $p$ plants at a nursery for $5.99 each. The total he paid was $65.89.

113. Quarters The number of cents in $q$ quarters is 175.

114. Seconds The number of seconds in $m$ minutes is 480.

115. Age Darta Aguilar is 1 year older than twice Julie Chesser's age. The sum of their ages is 52.

116. Horses Marc Campbell owns more horses than Selina Jones owns. The number of horses that Marc owns is 3 less than twice the number of horses Selina owns.

112. Plants Mark Ernsthausen purchased $p$ plants at a nursery for $5.99 each. The total he paid was $65.89.

\[ p = \frac{11}{5.99} \]
99. **Two Numbers** One number is 4 times another. The sum of the two numbers is 20.

100. **Age** Marie is 6 years older than Denise. The sum of their ages is 48.

\[
\begin{align*}
\text{Sum of Ages is 48} \\
\text{Marie:} & \quad x \\
\text{Denise:} & \quad x - 6
\end{align*}
\]

\[
\begin{align*}
2x - 6 &= 48 \\
2x &= 54 \\
x &= 27
\end{align*}
\]

\[
\begin{align*}
x - 6 &= 21
\end{align*}
\]
In Example 10 we will use the term **consecutive even integers**. **Consecutive integers** are integers that differ by 1 unit. For example, the integers 6 and 7 are consecutive integers. Two consecutive integers may be represented as $x$ and $x + 1$. **Consecutive even integers** are even integers that differ by 2 units. For example, 6 and 8 are consecutive even integers. **Consecutive odd integers** also differ by 2 units. For example, 7 and 9 are consecutive odd integers. Two consecutive even integers or two consecutive odd integers may be represented as $x$ and $x + 2$, where $x$ is always the smaller of the integers and $x + 2$ is always the larger of the integers.

**EXAMPLE 10**  
**Consecutive Even Integers** Write the problem as an equation.

For two consecutive even integers, the sum of the smaller and 3 times the larger is 22.

**Solution** First, we express the two consecutive even integers in terms of the variable.

Let $x =$ smaller consecutive even integer.

Then $x + 2 =$ larger consecutive even integer.

Now we write the equation using the information given.

$$\text{smaller} + 3 \times \text{the larger} = 22$$

Now Try Exercise 107

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102. **Even Integers** The product of two consecutive even integers is 74.

103. **Numbers** Twice a number, decreased by 8 is 12.

104. **Consecutive Integers** For two consecutive integers, the sum of the smaller and twice the larger is 29.

105. **Numbers** One-fifth of the sum of a number and 10 is 150.

106. **Numbers** One-third of the sum of a number and 12 is 5.

107. **Even Integers** For two consecutive even integers, the sum of the smaller and twice the larger is 22.

108. **Odd Integers** For two consecutive odd integers, the sum of 3 times the smaller and the larger is 14.

109. **Earnings** John Jones earns $12.50 per hour. If he works $h$ hours his earnings will be $150.

110. **Employees** The T.W. Wilson company plans to increase its number of employees by 20 per year. The increase in the number of employees in $t$ years will be 120.

111. **Top Soil** Abe Mantell purchased $x$ bags of top soil at a cost of $2.99 a bag. The total he paid was $17.94.

112. **Plants** Mark Ernsthausen purchased $p$ plants at a nursery for $5.99 each. The total he paid was $65.89.

113. **Quarters** The number of cents in $q$ quarters is 125.