23. \((0, \frac{5}{2})\) and \((-\frac{3}{4}, 2)\)  

\(Slope = m = \frac{y_2 - y_1}{x_2 - x_1}\)  

\[m = \frac{\frac{5}{2} - 2}{0 - (-\frac{3}{4})} = \frac{\frac{5}{2} - \frac{4}{2}}{\frac{3}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}\]  

\[\frac{0.5}{\frac{5}{2}} = \frac{5}{2} = \frac{5}{3} = \frac{2}{3}\]  

\[0.666 = \frac{2}{3}\]
Formula: \( m = \frac{3 - (-3)}{-4 - (-1)} = \frac{6}{3} = \frac{-2}{1} \)

\[ m = \frac{-6}{3} = \frac{-2}{1} \]
31. \[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{4} = \frac{1}{2}\]

Positive Slope

\[m = \frac{\text{rise}}{\text{run}} = \frac{2}{4}\]

\[m = \frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}\]
In Exercises 35–48, graph the line with the given slope that goes through the given point.

35. Through (3, –1) with \( m = 2 \).
36. Through (−1, −2) with \( m = -2 \).
37. Through (0, −2) with \( m = \frac{1}{2} \).
38. Through (0, 0) with \( m = -1 \).
39. Through (2, 3) with \( m = -\frac{1}{3} \).
40. Through (–3, 1) with \( m = \frac{2}{3} \).

\[ m = \frac{-2-(-2)}{-2-(-2)} = 0 \]

Undefined Slope

No Slope
41. Through (0, -2) with $m = \frac{1}{2}$

\[
\frac{\text{Rise}}{\text{Run}} = \frac{\text{Vertical Change}}{\text{Horizontal Change}}
\]

\[
\frac{-1}{-2}
\]
40. Through \((-1, -2)\) with \(m = -2\)
\[ M = 0 \]
\[ (2, y) \]

\[ y = 4 \]

Equation

Horizontal Line
Vertical Line
Undefined Slope

$(-1, 3)$

Equation

$x = -1$
7.4 Slope-Intercept and Point-Slope Forms of a Linear Equation

In Section 7.1 we introduced the standard form of a linear equation, \( ax + by = c \). In this section we introduce two more forms, the slope-intercept form and the point-slope form. We begin our discussion with the slope-intercept form.

1 Write a Linear Equation in Slope-Intercept Form

A very important form of a linear equation is the slope-intercept form, \( y = mx + b \) (recall that we briefly discussed the \( y = mx + b \) form in Section 2.6). The graph of an equation of the form \( y = mx + b \) will always be a straight line with a slope of \( m \) and a y-intercept \((0, b)\). For example, the graph of the equation \( y = 3x - 4 \) will be a straight line with a slope of 3 and a y-intercept \((0, -4)\). The graph of \( y = -2x + 5 \) will be a straight line with a slope of \(-2\) and a y-intercept \((0, 5)\).

\[
\frac{2x + 3y}{2} = 6 \\
\frac{3y}{3} = -2x + c \\
\frac{y}{3} = -\frac{2}{3}x + 2
\]

\[
(0, 2) \quad y-intercept \\
(0, -6) \quad (0, 3)
\]

Slope-Intercept Form of a Linear Equation

\[
y = mx + b
\]

where \( m \) is the slope, and \((0, b)\) is the y-intercept of the line.

Equations in Slope-Intercept Form

<table>
<thead>
<tr>
<th>Equation</th>
<th>Slope</th>
<th>y-Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 4x - 6 )</td>
<td>4</td>
<td>(0, -6)</td>
</tr>
<tr>
<td>( y = \frac{1}{2}x + \frac{3}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( (0, \frac{3}{2}) )</td>
</tr>
<tr>
<td>( y = -5x + 3 )</td>
<td>-5</td>
<td>(0, 3)</td>
</tr>
</tbody>
</table>

Writing an Equation in Slope-Intercept Form

To write a linear equation in slope-intercept form, solve the equation for \( y \).
\[ 2x + 3y = 6 \]

\[ \frac{3y}{3} = \frac{-2x+6}{3} \]

\[ y = \frac{-2}{3}x + 2 \]

Plot points:

\[ \begin{array}{c|c|c}
    x & y & \text{Plot point} \\
    \hline
    0 & 2 & \text{(0,2)} \\
    3 & 0 & \text{(3,0)} \\
    -3 & 4 & \text{Point} \\
    6 & -2 & \text{Point} \\
    -6 & 2 & \text{Point} \\
\end{array} \]

Slope-intercept form:

\[ y = \frac{-2}{3}x + 2 \]

Slope = \( m \)

\[ m = \text{slope} \]

\[ (0, 0) \rightarrow y = \text{Intercept} \]
Determine the slope and y-intercept of the line represented by the given equation.

12. \(7x = 5y + 25\)

\[
\frac{5y}{5} = \frac{7x - 25}{5}
\]

\[
y = \frac{7}{5}x - 5
\]

\(m = \frac{7}{5}\)  
(0, 5)

20. \(3x + 3y = 9\)
20. $3x + 3y = 9$

$$3x + 3y = 9$$

$$\frac{3y}{3} = -3x + \frac{9}{3}$$

$$y = -\frac{3x}{3} + 3$$

$$y = -x + 3$$

$$y = mx + b$$

Slope $m$: $-$ Intercept $b$

$m = -1 = \frac{-1}{1}$

$(0,3)$

$m = \frac{1}{-1}$

$(1,2)$

$(1,4)$

$(0,3)$
2. **Graph a Linear Equation Using the Slope and y-Intercept**

In Section 7.2 we discussed two methods of graphing a linear equation. They were (1) by plotting points and (2) using the x- and y-intercepts. Now we present a third method. This method makes use of the slope and the y-intercept. Remember that when we solve an equation for y we put the equation in slope-intercept form. Once it is in this form, we can determine the slope and y-intercept of the graph from the equation.

We graph equations using the slope and y-intercept in a manner very similar to the way we worked Examples 5 and 6 in Section 7.3. However, when graphing using the slope-intercept form, our starting point is always the y-intercept. After you determine the y-intercept, a second point can be obtained by moving up and to the right if the slope is positive, or down and to the right if the slope is negative.

22. \(-x + 2y = 8\)

\[
\begin{align*}
2y &= x + 8 \\
\Rightarrow y &= \frac{1}{2}x + 4
\end{align*}
\]

\[m = \frac{1}{2}, \quad b = 4\]
22. \(-x + 2y = 8\)
3 Use the Slope-Intercept Form to Determine the Equation of a Line

Now that we know how to use the slope-intercept form of a line, we can use it to write the equation of a given line. To do so, we need to determine the slope, \( m \), and \( y \)-intercept of the line. Once we determine these values we can write the equation in slope-intercept form, \( y = mx + b \). For example, if we determine the slope of a line is \(-4\) and the \( y \)-intercept is at \( 6 \), the equation of the line is \( y = -4x + 6 \).

**Determine the equation of each line.**
Determine the equation of each line.

32.

\[ y = mx + b \]

\[ y = \text{-intercept} (0, -2) \]

\[ m = \frac{-2}{4} = -\frac{1}{2} \]

\[ = -\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2} \]

\[ y = -\frac{1}{2}x - 2 \]
$y = mx + b$

$(b, 2.5)$

$y$-intercept
66. **Submarine Submerges** A submarine is submerged below sea level. Tom Johnson, the captain, orders the ship to dive slowly. The following graph illustrates the submarine's depth at a time $t$ minutes after the submarine begins to dive.
4 Use the Point-Slope Form to Determine the Equation of a Line

Thus far, we have discussed the standard form of a linear equation, $ax + by = c$, and the slope-intercept form of a linear equation, $y = mx + b$. Now we will discuss another form, called the point-slope form.

When the slope of a line and a point on the line are known, we can use the point-slope form to determine the equation of the line. The point-slope form can be obtained by beginning with the slope between any selected point $(x, y)$ and a fixed point $(x_1, y_1)$ on a line.

$$m = \frac{y - y_1}{x - x_1}$$

Now cross-multiply to obtain

$$m(x - x_1) = y - y_1 \text{ or } y - y_1 = m(x - x_1)$$

**Point-Slope Form of a Linear Equation**

$$y - y_1 = m(x - x_1)$$

where $m$ is the slope of the line and $(x_1, y_1)$ is a point on the line.

Write the equation of each line, with the given properties, in slope-intercept form.

54. Slope $= -\frac{2}{3}$, through $(4, -5)$

$$m = -\frac{2}{3} \text{ or } \frac{y - y_1}{x - x_1} = \frac{-2}{3}$$

$$y - (-5) = \left(-\frac{2}{3}\right)(x - 4)$$

$$y + 5 = \left(-\frac{2}{3}\right)x + \frac{8}{3}$$

$$3y + 15 = -2x + 8$$

$$2y = -2x + 8$$

$$\frac{2}{3}x + \frac{3}{2}y = \frac{8}{3}$$

$$2x + 3y + 15 = 8$$

$$-15 - 15$$

$$\frac{2}{3}x + \frac{3}{2}y = -\frac{7}{3}$$

Slope-Intercept Form

$$3y + 15 = -2x + 8$$

$$\frac{3}{5} = -\frac{2}{5}x - \frac{7}{3}$$

$$y = -\frac{2}{5}x - \frac{7}{3}$$
Write the equation of each line, with the given properties, in slope-intercept form.

56. Slope = $\frac{1}{9}$, y-intercept is $(0, -\frac{2}{3})$

\[ y = mx + b \]
\[ y - y_1 = m(x - x_1) \]
\[ y = \frac{1}{9}x - \frac{2}{3} \]

Point + Slope:
\[ y - y_1 = m(x - x_1) \]
\[ y - \left(-\frac{2}{3}\right) = \frac{1}{9}(x - 0) \]
\[ y + \frac{2}{3} = \frac{1}{9}x \]
\[ y = \frac{1}{9}x - \frac{2}{3} \]
**Helpful Hint**

We have discussed three forms of a linear equation. We summarize the three forms below. It is important that you memorize these forms.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax + by = c$</td>
<td>$2x - 3y = 8$</td>
</tr>
<tr>
<td></td>
<td>$-5x + y = -2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slope-Intercept Form</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = mx + b$</td>
<td>$y = 2x - 5$</td>
</tr>
<tr>
<td>$m$ is the slope, $(0, b)$ is the y-intercept</td>
<td>$y = -\frac{3}{2}x + 2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point-Slope Form</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y - y_1 = m(x - x_1)$</td>
<td>$y - 3 = 2(x + 4)$</td>
</tr>
<tr>
<td>$m$ is the slope, $(x_1, y_1)$ is a point on the line</td>
<td>$y + 5 = -4(x - 1)$</td>
</tr>
</tbody>
</table>
We now discuss how to use the point-slope form to determine the equation of a line when two points on the line are known.

58. Through (7, 4) and (6, 3)

\[ y - y_1 = m(x - x_1) \]

1) Find slope

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{7 - 6} = 1 \]

Choose point \( M = \) \( (6, 3) \)

Put into point slope equation

\[ y - y_1 = m(x - x_1) \]

\[ y - 3 = 1(x - 6) \]

\[ y - 3 = x - 6 \]

Solve for \( y \)

\[ y = x - 3 \]
We now discuss how to use the point-slope form to determine the equation of a line when two points on the line are known.

62. Through \((-6, -2)\) and \((5, -3)\)
**Helpful Hint**

In the exercise set at the end of this section, you will be asked to write a linear equation in slope-intercept form. Even though you will eventually write the equation in slope-intercept form, you may need to start your work with the point-slope form. Below we indicate the initial form to use to solve the problem.

Begin with the **slope-intercept form** if you know

The slope of the line and the y-intercept

Begin with the **point-slope form** if you know

a) The slope of the line and a point on the line, or
b) Two points on the line (first find the slope, then use the point-slope form)

---

**Example 10**

24. $16y = 8x + 32$

a) by plotting points;
b) using the x- and y-intercepts;
c) using the slope and y-intercept.