1.9 Exponents, Parentheses, and the Order of Operations

1. Learn the Meaning of Exponents

In the expression $4^2$, the 4 is called the base, and the 2 is called the exponent. The number $4^2$ is read "4 squared" or "4 to the second power" and means

\[ 4 \cdot 4 = 4^2 \]

(base) \hspace{1cm} \text{exponent}

2 factors of 4

The number $4^3$ is read "4 cubed" or "4 to the third power" and means

\[ 4 \cdot 4 \cdot 4 = 4^3 \]

3 factors of 4

In general, the number $b$ to the $n$th power, written $b^n$, means

\[ b \cdot b \cdot \ldots \cdot b = b^n \]

$n$ factors of $b$

Thus, $b^4 = b \cdot b \cdot b \cdot b$ or $bbbb$ and $x^3 = x \cdot x \cdot x$ or $xxx$.

2. Evaluate Expressions Containing Exponents

Let's evaluate some expressions that contain exponents.

**EXAMPLE 1** Evaluate. a) $3^2$ b) $2^5$ c) $1^5$ d) $(-6)^2$ e) $(-2)^3$ f) $\left(\frac{2}{3}\right)^2$

\[ 3^2 = 3 \cdot 3 = 9 \]
\[ 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 \]
\[ 1^5 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1 \]
\[ (-6)^2 = (-6) \cdot (-6) = 36 \]
\[ (-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8 \]
\[ \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right) = \frac{4}{9} \]
It is not necessary to write exponents of 1. For example, when writing $xxy$, we write $x^2y$ and not $x^2y^1$. Whenever we see a variable or number without an exponent, we always assume that the variable or number has an exponent of 1.

**Examples of Exponential Notation**

a) $xyx = x^2y$

b) $xyzxy = xy^2z^2$

c) $3aabb = 3a^2b^3$

d) $5xyyy = 5xy^4$

e) $4 \cdot 4rrs = 4^2r^2s$

f) $5 \cdot 5 \cdot 5nnn = 5^3m^2n$

Notice in parts a) and b) that the order of the factors does not matter.

**Helpful Hint**

Note that $x + x + x + x + x = 6x$ and $x \cdot x \cdot x \cdot x \cdot x = x^5$. Be careful that you do not get addition and multiplication confused.
3 Learn the Difference Between $-x^2$ and $(-x)^2$

An exponent refers only to the number or variable that directly precedes it unless parentheses are used to indicate otherwise. For example, in the expression $3x^2$, only the $x$ is squared. In the expression $-x^2$, only the $x$ is squared. We can write $-x^2$ as $-1x^2$ because any real number may be multiplied by 1 without affecting its value.

$$-1x^2 = -x^2$$

By looking at $-1x^2$ we can see that only the $x$ is squared, not the $-1$. If the entire expression $-x$ were to be squared, we would need to use parentheses and write $(-x)^2$. Note the difference in the following two examples:

$$-x^2 = -(x)(x)$$

$$(-x)^2 = (-x)(-x)$$

Consider the expressions $-3^2$ and $(-3)^2$. How do they differ?

$$-3^2 = -(3)(3) = -9$$

$$(-3)^2 = (-3)(-3) = 9$$

**EXAMPLE 2** Evaluate.  

<table>
<thead>
<tr>
<th></th>
<th>a) $-5^2$</th>
<th>b) $(-5)^2$</th>
<th>c) $-2^3$</th>
<th>d) $(-2)^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-5^2$</td>
<td>$-(5)(5) = -25$</td>
<td>$(-5)^2 = (-5)(-5) = 25$</td>
<td>$-2^3 = -(2)(2)(2) = -8$</td>
<td>$(-2)^3 = (-2)(-2)(-2) = -8$</td>
</tr>
<tr>
<td>$-(-2)^2$</td>
<td>$(-1)(-2)(-2) = -4$</td>
<td>$(-2)^3 = (-2)(-2)(-2) = -8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-3)^3$</td>
<td>$(-3)(-3)(-3) = -27$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLE 3** Evaluate.  

<table>
<thead>
<tr>
<th></th>
<th>a) $-2^4$</th>
<th>b) $(-2)^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2^4$</td>
<td>$-(2)(2)(2)(2)$</td>
<td>$(-2)^4 = (2)(2)(2)(2) = 16$</td>
</tr>
</tbody>
</table>
Learn the Order of Operations

Now that we have introduced exponents we can present the order of operations. Can you evaluate $2 + 3 \cdot 4$? Is it 20? Or is it 14? To answer this, you must know the order of operations to follow when evaluating a mathematical expression.

**Order of Operations:** To Evaluate Mathematical Expressions, Use the Following Order

1. First, evaluate the information within **parentheses** ( ), brackets [ ], or braces { }. These are **grouping symbols**, for they group information together. A fraction bar, $\frac{1}{2}$, also serves as a grouping symbol. If the expression contains nested grouping symbols (one pair of grouping symbols within another pair), evaluate the information in the inner-most grouping symbols first.
2. Next, evaluate all **exponents**.
3. Next, evaluate all **multiplications or divisions** in the order in which they occur, working from left to right.
4. Finally, evaluate all **additions or subtractions** in the order in which they occur, working from left to right.

Some students remember the word PEMDAS or the phrase "Please Excuse My Dear Aunt Sally" to help them remember the order of operations. PEMDAS helps them remember the order: Parentheses, Exponents, Multiplication, Division, Addition, Subtraction. Remember, this does not imply multiplication before division or addition before subtraction.
66. $-2 + 4[-3 + (48 \div 4^2)]$

\[\downarrow\]

\[-2 + 4[-3 + 12^2]\]

\[-2 + 4[-3 + 144]\]

\[-2 + 4[141]\]

\[-2 + 564\]

\[562\]

72. $(12 \div 4) + 5(6 - 4)^2$
70. \(-7 - 56 \div 7 \cdot 2^2 + 4\)  

76. \(-3^3 + 8 \div 2\)
104. $2x - 4x + 5; \ x = 3$

Evaluate a) $x^2$, b) $-x^2$, and c) $(-x)^2$ for the following values of $x$.

95. 5 96. 8 97. $-2$ 98. $-5$
112. \(-t^2 - 4t + 5; t = -4\)

120. \(4(x + y)^2 + 2(x + y) + 3; x = 2, y = 4\)