4.4 Addition and Subtraction of Polynomials

Identify Polynomials

A polynomial in \( x \) is an expression containing the sum of a finite number of terms of the form \( ax^n \), for any real number \( a \) and any whole number \( n \).

<table>
<thead>
<tr>
<th>Examples of Polynomials</th>
<th>Not Polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x</td>
<td>( 4x^{\frac{1}{2}} ) (Fractional exponent)</td>
</tr>
<tr>
<td>( \frac{1}{3} x - 4 )</td>
<td>( 3x^2 + 4x^{-1} + 5 ) (Negative exponent)</td>
</tr>
<tr>
<td>( x^2 - 2x + 1 )</td>
<td>( 4 + \frac{1}{x} ) (( \frac{1}{x} = x^{-1} ), negative exponent)</td>
</tr>
</tbody>
</table>

A polynomial is written in **descending order** (or descending powers) of the variable when the exponents on the variable decrease from left to right.

**Example of Polynomial in Descending Order**

\[ 2x^4 + 4x^3 - 6x + 3 \]

Note in the example that the constant term 3 is last because it can be written as \( 3x^0 \). Remember that \( x^0 = 1 \).

A polynomial can be in more than one variable. For example, \( 3xy + 2 \) is a polynomial in two variables, \( x \) and \( y \).

A polynomial with one term is called a **monomial**. A **binomial** is a two-termed polynomial. A **trinomial** is a three-termed polynomial. Polynomials containing more
<table>
<thead>
<tr>
<th>Type of Polynomial</th>
<th>Number of Terms</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomial</td>
<td>One</td>
<td>8, 4x, -6x²</td>
</tr>
<tr>
<td>Binomial</td>
<td>Two</td>
<td>x + 5, x² - 6, 4y² - 5y</td>
</tr>
<tr>
<td>Trinomial</td>
<td>Three</td>
<td>x² - 2x + 3, 3z² - 6z + 7</td>
</tr>
</tbody>
</table>

The **degree of a term** of a polynomial in one variable is the exponent on the variable in that term.

<table>
<thead>
<tr>
<th>Term</th>
<th>Degree of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>4x²</td>
<td>Second</td>
</tr>
<tr>
<td>2y⁵</td>
<td>Fifth</td>
</tr>
<tr>
<td>-5x</td>
<td>First</td>
</tr>
<tr>
<td>3</td>
<td>Zero</td>
</tr>
</tbody>
</table>

(−5x can be written −5x.)

(3 can be written 3x².)

For a polynomial in two or more variables, the degree of a term is the sum of the exponents on the variables. For example, the degree of the term 4x²y³ is 5 because 2 + 3 = 5. The degree of the term 5x⁴y⁶z is 8 because 4 + 1 + 3 = 8.

The **degree of a polynomial** is the same as that of its highest-degree term.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree of Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x³ + 2x³ - 3x + 4</td>
<td>Third (8x³ is highest-degree term.)</td>
</tr>
<tr>
<td>x² - 4</td>
<td>Second (x² is highest-degree term.)</td>
</tr>
<tr>
<td>6x - 5</td>
<td>First (6x or 6x¹ is highest-degree term.)</td>
</tr>
<tr>
<td>4</td>
<td>Zero (4 or 4x⁰ is highest-degree term.)</td>
</tr>
<tr>
<td>x²y⁴ + 2x + 3</td>
<td>Sixth (x²y⁴ is highest-degree term.)</td>
</tr>
</tbody>
</table>

**Indicate the degree of each term.**

19. −3h⁶  =  Degree 6 = Degree 24. 6m²n³  =  Degree 4

**Indicate which expressions are polynomials. If the polynomial has a specific name—monomial, binomial, or trinomial—give that name.**

31. 9a⁴ - 5
34. 3x² + 2x
37. 6n³ - 5n² + 4n - 3
40. 5p⁷

**Express each polynomial in descending order. If the polynomial is already in descending order, so state. Give the degree of each polynomial.**

50. 2x² + 5x - 8
53. −4 + x - 3x² + 4x³
2 Add Polynomials

In Section 2.1, we stated that like terms are terms having the same variables and the same exponents. That is, like terms may differ only in their numerical coefficients.

**Examples of Like Terms**

- $3$, $-5$
- $2x$, $x$
- $-2x^2$, $4x^2$
- $3y^3$, $5y^3$
- $3xy^2$, $5xy^2$

**To Add Polynomials**

To add polynomials, combine the like terms of the polynomials.

**EXAMPLE 1** Add $(4x^2 + 6x + 3) + (2x^2 + 5x - 1)$.

**Solution** Remember that $(4x^2 + 6x + 3) = 1(4x^2 + 6x + 3)$ and $(2x^2 + 5x - 1) = 1(2x^2 + 5x - 1)$. We can use the distributive property to remove the parentheses, as shown below.

\[
(4x^2 + 6x + 3) + (2x^2 + 5x - 1) \\
= 1(4x^2 + 6x + 3) + 1(2x^2 + 5x - 1) \\
= 4x^2 + 6x + 3 + 2x^2 + 5x - 1 \\
= 4x^2 + 2x^2 + 6x + 5x + 3 - 1 \\
]\]

**Add Like Terms**

- $5x - 9$
- $-9x$
- $7x - 6$
- $-3p^2 - 3p - 6$
- $-3p^2 - 3p - 6$

**Exercise**

- **58.** $(5x - 6) + (2x - 3)$
- **60.** $(-7x - 9) + (-2x + 9)$
- **62.** $(4x - 3) + (3x - 3)$
- **64.** $(-4p^2 - 3p - 2) + (-2p^2 - 4p)$

- **96.** $(9x^2 + 7x - 5) - (3x^2 + 3.5)$
- **98.** $(-p^2 + 5x + 12) - (-4p^2 - 3)$
- **100.** $(7x - 6) - (-2x^2 + 4x - 8)$

- **106.** Subtract $(-4x + 7)$ from $(-3x - 9)$.
- **108.** Subtract $(3x^2 - 5x - 3)$ from $(-x^2 + 3x + 10)$. 

- **109.** Subtract $(-x^2 + 3x + 10)$ from $(3x^2 - 5x - 3)$.
\[ x^2 + x^2 + x^2 = 3x^2 \]
\[ \text{Area of Blue} = 3xy \]
\[ \text{Total Area} = 3x^2 + 3xy \]