(1) \[ x^5 \cdot x^3 = x^8 \]

(2) \[ \frac{a^7}{a^2} = a^{7-2} = a^5 \]

(3) \[ (3x^3y^2)^3 = (3)^3 (x^3)^3 (y^2)^3 = 27x^9y^6 \]

(4) \[ (5x^2y^{-2}) (-2x^{-1}y^3) = -10x^{2-1}y^{-2+3} = \frac{-10x^1}{y} \]

(5) \[ (-1x^{-1}y^2) (-3x^{-1}y^3) = 3x^{-1-1}y^{-2+3} = \frac{3y^1}{x^2} \]
\[ (2x^{-4}y)^{-2} = (2)^{-2} (x^{-4})^{-2} (y)^{-2} \]

\[ 2^{-2} = \frac{1}{4} = \frac{1}{2^2} \]

\[ \frac{1^{10} y^{-3}}{5x^{-2} y^4} = \frac{x^{12} y^{-7}}{5} = \frac{x^{12}}{5y^7} \]

\[ \frac{x^{10}}{x^{-2}} = x^{10 - (-2)} = x^{12} \]

\[ \frac{y^{-3}}{y^4} = y^{-3 - 4} = y^{-7} \]
\[
\frac{(3.82 \times 10^{10}) (5.0 \times 10^{13})}{3.2 \times 10^{-6}} = \frac{19.482 \times 10^{35}}{3.2 \times 10^{-6}} = 5.96 \times 10^7
\]

\[
5x^2y^3z^2
\]

Degree of Each Term:
- 9
- 2
- 3
- 1

Degree of Polynomial:
Highest Degree Term = 10
15 \quad (4x^2 - 3x + 4) + (x^2 - 2x - 2)

\quad 5x^2 - 5x - 3

16 \quad (4x^2 - 5) + (x^2 - 3) = 5x^2 - 8

17 \quad (2x^3 - 3x - 5) + (4x^2 + 2x + 6)

\quad 2x^3 - 4x^2 - x - 15
18. \((5b-7)-3(3b+1)\)
\[5b - 7 - 9b - 3\]
\[-4b - 10\]

19. \((2x-3)(4x^2 - 3x - 2)\)
Multiplying:
\[8x^3 - 6x^2 - 4x\]
\[-12x^2 + 9x + 6\]
\[8x^3 - 18x^2 + 5x + 6\]
20. \((4x - 3)(3x + 4)\)

\[12x^2 + 16x - 9x - 12\]

\[12x^2 + 7x - 12\]

21. \((10x - 7)(10x + 7)\)

\[100x^2 + 70x - 70x - 49\]

\[100x^2 - 49\]
\[ (2x-1)(7x+4) \]

\[ 6x^2 + 8x - 3x - 4 \]

\[ 6x^2 + 5x - 4 \]

\[ (4x-5)^2 = (4x-5)(4x-5) \]

\[ 16x^2 - 20x - 20x + 25 \]

\[ 16x^2 - 40x + 25 \]
\((2x^2 - 1)(3x^3 - 4x + 5)\)

\[
\begin{array}{c}
6x^5 - 8x^3 + 10x^2 \\
-3x^3 + 6 + 4x - 5
\end{array}
\]

\[6x^5 - 11x^3 + 10x^2 + 4x - 5\]

\[\frac{12x^2}{2x} + \frac{6x}{2x} + \frac{4}{2x}\]

\[6x + 3 + \frac{2}{x}\]
\[ D = R \cdot T \]

<table>
<thead>
<tr>
<th></th>
<th>120</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000-d</td>
<td>d</td>
<td>t</td>
</tr>
</tbody>
</table>

\[ d = 120t \]

\[ 2000 - d = 65t \]

\[ 2000 = 120t + 65t \]
\[
\frac{5x + 7}{2x - 3} + \frac{2}{2x^3} \div \frac{10x^2 - x - 19}{10x^2 + 15x} - \frac{14x - 19}{(19 - 21)}
\]
\[
\begin{array}{c|c|c|c}
\text{Amount} & \text{Rate} & \text{Interest} \\
\hline
x & .06 & .06x \\
\hline
x + 1000 & .09 & .09(x + 1000) \\
\hline
2x + 1000 & & 690 \\
\end{array}
\]

\[.06x + .09(x + 1000) = 690\]
\[.06x + .09x + 90 = 690\]
\[
\frac{.15x}{.15} = \frac{600}{.15}
\]
\[x + 1000 = 5000\]
\[x = 4000\]
1. \( (x^3)^{-4} = x^{-12} = \frac{1}{x^{12}} \)

2. \( \frac{x^2}{x^{-2}} = x^{2-(-2)} = x^4 = x^9 \)

3. \( (-y)^{-2} = \frac{1}{(-y)^2} = \frac{1}{16} \)

4. \( (-y x^2)^{-2} = \left(\frac{1}{-y x^2}\right)^2 = \frac{1}{16 x^4} \)
\[ (2x^5y^{-4})^{-3} = \frac{(2)^3 (x^5)^{-3} (y^{-4})^{-3}}{8} = \frac{y^{12}}{x^{15}} \]
(6) \[ (8x^2y) \left( 5x^3y^{-4} \right) = 40x^5y^0 = 40x^5 \]
\[ y^4 \cdot y^{-4} = y^0 = 1 \]

(7) \[ \frac{32x^3y^{-4}}{4x^{-2}} = \frac{8x^5y^{-4}}{1} = \frac{8x^5}{y^4} \]
\[ \frac{x^3}{x^{-2}} = x^{3-(-2)} = x^{3+2} = x^5 \]
\[ y^0 = \]
\[ (9) \quad \left( \frac{6x^4 y^7}{z^3} \right)^{-2} = \frac{6^{-2} (x^4 \cdot x^7)}{(z^3)^2} y^{14} \]

\[ = \frac{x^{-8} y^{14}}{6^2 z^{-6}} = \frac{y^{14}}{36 x^8} \]

\[ (9) \quad \left( \frac{\rho^7 q^{-4}}{5 \rho^9} \right)^2 = \frac{(\rho^7)^2 (q^{-4})^2}{(5 \rho^9)^2} = \frac{\rho^{14} q^{-8}}{25 \rho^{18}} \]

\[ = \frac{1}{25 \rho^4 q^8} \]
$3^0 - 4^{-1} =
1 - \frac{1}{4} = 1 - \frac{1}{4} = \frac{3}{4}$
11. \[ 0.0000726 = 7.26 \times 10^{-5} \]

\[ 7.26 \times \frac{1}{10^5} \]

12. \[ 2.8 \times 10^5 = 280,000 \]

13. \[ (1.8 \times 10^{-9})(1.92 \times 10^5) = 3.096 \times 10^{-2} \]

\[ 10^9 \times 10^5 = 10^{14} \]

\[ 0.03096 \]

14. \[ \frac{7.8 \times 10^{-4}}{6.0 \times 10^{-2}} = 1.3 \times 10^{-2} = 0.013 \]

-4.(-2) = -2
\[ \frac{0.0035}{0.00062} = \frac{3.5 \times 10^{-3}}{2.6 \times 10^{-6}} = \frac{1.75}{10^{-3+6}} = 1.75 \times 10^3 \]

\[ -8 \rho^2 \sigma^4 r^{-1} \quad \text{Degree} = 9 \]

2+4+1 = 7
\( b^{-2} + 7 \)  
Not A Polynomial
Can't Have A Negative Exponent

18.

\[ 4x + 3x^3 - 7 - 3x^4 \]
\[-3x^4 + 3x^3 + 4x - 7 \quad \text{Descending Order} \]
Degree = 4

19.

\[ (-6x+7) + C - 6x^2 + 6x - 8 \]
\[-6x^2 + 0x - 1 = -6x^2 - 1\]
\[ (5x^2 + 2x - 5) + (x^2 + \frac{5}{2}x + 2) = 6x^2 + \frac{11}{2}x - 3 \]

\[ (6x^2 - 3x - 2) + (+4x^2 + 5x + 5) = 10x^2 + 2x + 3 \]

\[ (5x^3 - \frac{3}{2}x^2 + 8x - 1) + (-5x^2 + \frac{3}{2}x^2 + 5) = 5x^3 - 3\frac{1}{3}x^2 + 3x - 2 \]

\[ -\frac{3}{3} = -1 \]
23. \((-7x^3y^2)(7x^4y^4) = -49x^7y^6\)
   \[x^3 \cdot x^4 = x^7\]

24. 
   \(-6y^2(3y^2 + 4y - 2)\)
   \[-18y^4 - 24y^3 + 12y^2\]

25. 
   \((7x - 9)(3x + 2)\)
   \[21x^2 + 14x - 27x - 18\]
   \[21x^2 - 13x - 18\]
24 \quad (5x-1)(x-5) 
\quad -5x^2 + 25x - 1x + 5 
\quad -5x^2 + 24x + 5 

27 \quad (2x-3)(2x+3) 
\quad 4x^2 + 6x - 6x - 9 
\quad 4x^2 - 9 

Conjugates

28 \quad (2x-5)(2x-5) = (2x-5)^2 
\quad (a-b)^2 = a^2 - ab - ab + b^2 
\quad = a^2 - 2ab + b^2 
\quad 4x^2 - 10x - 10x + 25 
\quad 4x^2 - 20x + 25
(29) \((x+3)(x+3) = (x+3)^2\)

\[
16x^2 + 12x + 12x + 9
\]

\[
16x^2 + 24x + 9
\]

(30) \((2x-3y)^2 = (2x-3y)(2x-3y)\)

\[
49x^2 - 21xy - 21xy + 9y^2
\]

\[
49x^2 - 42xy + 9y^2
\]
\[ (3a+9)(5a^2+9a+2) \]

\[ 15a^3 + 27a^2 + 6a \]

\[ 45a^2 + 81a + 18 \]

\[ 15a^3 + 72a^2 + 87a + 18 \]
\[ \text{Equation} \]
\[ 0.04x + 0.09(15000 - x) = 720 \]
\[ 0.04x + 1170 - 0.09x = 720 \]
\[ -0.05x + 1170 = 720 \]
\[ -0.05x = -450 \]
\[ x = 9000 \]
5.1 Factoring a Monomial from a Polynomial

1 Identify factors.
2 Determine the greatest common factor of two or more numbers.
3 Determine the greatest common factor of two or more terms.
4 Factor a monomial from a polynomial.

1 Identify Factors

In Chapter 4, you learned how to multiply polynomials. In this chapter, we focus on factoring, the reverse process of multiplication. In Section 4.5, we showed that $3x(2x^3 + 4) = 6x^4 + 12x$. In this chapter, we start with an expression like $6x^3 + 12x$ and determine that its factors are $3x$ and $2x^2 + 4$, and write $6x^3 + 12x = 3x(2x^2 + 4)$. To factor an expression means to write the expression as a product of its factors. Factoring is important because it can be used to solve equations and perform operations on fractions.

If $a \cdot b = c$, then $a$ and $b$ are said to be factors of $c$.

- $3 \cdot 5 = 15$; so 3 and 5 are factors of 15.
- $x^3 \cdot x^4 = x^7$; so $x^3$ and $x^4$ are factors of $x^7$.
- $(x + 2) = x^2 + 2x$; so $x$ and $x + 2$ are factors of $x^2 + 2x$.
- $(x - 1)(x + 3) = x^2 + 2x - 3$; so $x - 1$ and $x + 3$ are factors of $x^2 + 2x - 3$.

A given number or expression may have many factors. Consider the number 30:

$$1 \cdot 30 = 30, \quad 2 \cdot 15 = 30, \quad 3 \cdot 10 = 30, \quad 5 \cdot 6 = 30$$

So, the positive factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30. Factors can also be negative. Since $(-1)(-30) = 30$, $-1$ and $-30$ are also factors of 30. In fact, for each factor $a$ of an expression, $-a$ must also be a factor. Other factors of 30 are therefore $-1$, $-2$, $-3$, $-5$, $-6$, $-10$, $-15$, and $-30$. When asked to list the factors of an expression that contains a positive numerical coefficient with a variable, we generally list only positive factors.
EXAMPLE 1  ▶ List the factors of $6x^3$.

**Solution**

<table>
<thead>
<tr>
<th>Factors</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \cdot 6x^3 = 6x^3$</td>
<td>$x \cdot 6x^2 = 6x^3$</td>
</tr>
<tr>
<td>$2 \cdot 3x^3 = 6x^3$</td>
<td>$2x \cdot 3x^2 = 6x^3$</td>
</tr>
<tr>
<td>$3 \cdot 2x^3 = 6x^3$</td>
<td>$3x \cdot 2x^2 = 6x^3$</td>
</tr>
<tr>
<td>$6 \cdot x^3 = 6x^3$</td>
<td>$6x \cdot x^2 = 6x^3$</td>
</tr>
</tbody>
</table>

The factors of $6x^3$ are 1, 2, 3, 6, $x$, $2x$, $3x$, $6x$, $x^2$, $2x^2$, $3x^2$, $6x^2$, $x^3$, $2x^3$, $3x^3$, and $6x^3$. The opposite (or negative) of each of these factors is also a factor, but these opposites are generally not listed unless specifically asked for.

7. List the factors of $4x^2$.
Here are examples of multiplying and factoring. Notice again that factoring is the reverse process of multiplying.

<table>
<thead>
<tr>
<th>Multiplying</th>
<th>Factoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(2x + 5) = 6x + 15$</td>
<td>$6x + 15 = 3(2x + 5)$</td>
</tr>
<tr>
<td>$4y(y - 7) = 4y^2 - 28y$</td>
<td>$4y^2 - 28y = 4y(y - 7)$</td>
</tr>
<tr>
<td>$(x + 1)(x + 3) = x^2 + 4x + 3$</td>
<td>$x^2 + 4x + 3 = (x + 1)(x + 3)$</td>
</tr>
</tbody>
</table>

2 Determine the Greatest Common Factor of Two or More Numbers

To factor a monomial from a polynomial, we make use of the greatest common factor (GCF). If after studying the following material you wish to see additional material on obtaining the GCF, you may read Appendix B, where one of the topics discussed is finding the GCF.

Recall from Section 1.3 that the greatest common factor of two or more numbers is the greatest number that divides into all the numbers. The greatest common factor of the numbers 6 and 8 is 2. Two is the greatest number that divides into both 6 and 8. What is the GCF of 48 and 60? When the GCF of two or more numbers is not easily found, we can find it by writing each number as a product of prime numbers. A prime number is an integer greater than 1 that has exactly two factors, itself and one. The first 15 prime numbers are

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$$

A positive integer (other than 1) that is not prime is called composite. The number 1 is neither prime nor composite, it is called a unit. The first 15 composite numbers are

$$4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25$$

Every even number greater than 2 is a composite number since it has more than two factors, itself, 1, and 2.

To write a number as a product of prime numbers, follow the procedure illustrated in Examples 2 and 3.

In Example 2, we found that $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2^4 \cdot 3$. The $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ or $2^4 \cdot 3$ may also be referred to as prime factorizations of 48.
To Determine the GCF of Two or More Numbers

1. Write each number as a product of prime factors.
2. Determine the prime factors common to all the numbers.
3. Multiply the common factors found in step 2. The product of these factors is the GCF.

EXAMPLE 4

Determine the greatest common factor of 48 and 60.

Solution

From Examples 2 and 3, we know that

Step 1

\[
48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2^4 \cdot 3 \\
60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5
\]

Step 2

The common factors are circled. Two factors of 2 and one factor of 3 are common to both numbers. The product of these factors is the GCF of 48 and 60:

Step 3

GCF = \(2 \cdot 2 \cdot 3 = 12\)

The GCF of 48 and 60 is 12. Twelve is the greatest number that divides into both 48 and 60.

To Determine the Greatest Common Factor of Two or More Terms

To determine the GCF of two or more terms, take each factor the largest number of times that it appears in all of the terms.

EXAMPLE 8

Determine the GCF of the terms \(xy, x^2y^2, \) and \(x^3\).

Solution

The GCF is \(x\). The largest power of \(x\) that is common to all three terms is \(x^1\), or \(x\). Since the term \(x^3\) does not contain a power of \(y\), the GCF does not contain \(y\).

Now Try Exercise 25

16. \(45, 27\)

\[
\begin{array}{c|c}
45 & 27 \\
3 \cdot 15 & 3 \cdot 9 \\
3 \cdot 3 \cdot 5 & 3 \cdot 3 \\
\hline & 3 \\
\end{array}
\]

GCF = 3

22. \(y^3, y^5, y^2\)

\[
\begin{array}{c|c}
y^3 & y^5 \\
\hline & y^2 \quad \text{or} \quad y^\text{largest} \\
\end{array}
\]

28. \(4x^2y^2, 3xy^4, 2xy^2\)

\[
\begin{array}{c|c}
4x^2y^2 & 3xy^4, 2xy^2 \\
2 \cdot 2 \cdot x \cdot x \cdot y \cdot y & 2 \cdot 3 \cdot y \cdot y \cdot y \\
\hline & 1 \cdot 3 \cdot y \\
\end{array}
\]

GCF = \(2 \cdot 3 \cdot y = 6\)

30. \(6x, 12y, 18x^2\)

\[
\begin{array}{c|c}
6x & 12y \\
2 \cdot 3 \cdot x & 2 \cdot 2 \cdot 3 \cdot y \\
2 \cdot 3 \cdot 3 \\
\hline & 1 \cdot 3 \cdot 3 \cdot x \cdot y \\
\end{array}
\]

GCF = \(2 \cdot 3 = 6\)

40. \(x(9x - 3), 9x - 3\)

\[
\begin{array}{c|c}
x(9x - 3) & 9x - 3 \\
\hline & 9x - 3 \\
\end{array}
\]

GCF = \(9x - 3\)
4  Factor a Monomial from a Polynomial

In Section 4.5 we multiplied factors. Factoring is the reverse process of multiplying factors. As mentioned earlier, to *factor an expression* means to write the expression as a product of its factors.

**To Factor a Monomial from a Polynomial**

1. Determine the greatest common factor of all terms in the polynomial.
2. Write each term as the product of the GCF and its other factor.
3. Use the distributive property to factor out the GCF.

In step 3 of the process, we indicate that we use the distributive property. The distributive property is actually used in reverse. For example, if we have $4 \cdot x + 4 \cdot 2$, we use the distributive property in reverse to write $4(x + 2)$.

**EXAMPLE 11**  Factor $6x + 18$.

**Solution**  The GCF is 6.

\[
6x + 18 = 6 \cdot x + 6 \cdot 3 \\
= 6(x + 3)
\]

50.  $4x + 2$

   \[\boxed{2(2x+1)}\]

82.  $12x + 15$

   \[\boxed{3(4x+5)}\]

60.  $26p^2 - 8p$

   \[\boxed{2p(13p-4)}\]

82.  $12a^3 - 16a^2 - 4a$

   \[\boxed{4a(3a^2 - 4a - 1)}\]
90. $52x^2y^2 + 16xy^2 + 26z$

\[2 \left(26x^2y^2 + 8xy^2 + 13z\right)\]

96. $\frac{a b - a c}{a (b-c)}$

\[6 \text{ GCF} = 5m - 1\]

\[4m(5m - 1) - 3(5m - 1)\]

\[(5n-1)(4n-3)\]

100. $5(t - 2) - 3(t - 2)$

\[(t-2)(5t-3)\]