5.2 Factoring by Grouping

1 Factor a Polynomial Containing Four Terms by Grouping

It may be possible to factor a polynomial containing four or more terms by removing common factors from groups of terms. This process is called factoring by grouping. In Sections 5.3 and 5.4, we discuss factoring trinomials. One of the methods we will use in Section 5.4 requires a knowledge of factoring by grouping. Example 1 illustrates the procedure for factoring by grouping.

**Example 1**  
Factor $ax + ay + bx + by$ by grouping.

**Solution** There is no factor (other than 1) common to all four terms. However, $a$ is common to the first two terms and $b$ is common to the last two terms. Factor $a$ from the first two terms and $b$ from the last two terms.

$$ax + ay + bx + by = a(x + y) + b(x + y)$$

This factoring gives two terms, and $(x + y)$ is common to both terms. Proceed to factor $(x + y)$ from each term, as shown below.

$$a(x + y) + b(x + y) = (x + y)(a + b)$$

Notice that when $(x + y)$ is factored out we are left with $a + b$, which becomes the other factor. Thus, $ax + ay + bx + by = (x + y)(a + b)$.

8. $x^2 + 7x + 3x + 21$
To Factor a Four-Term Polynomial Using Grouping

1. Determine whether there are any factors common to all four terms. If so, factor the greatest common factor from each of the four terms.
2. If necessary, arrange the four terms so that the first two terms have a common factor and the last two have a common factor.
3. Use the distributive property to factor each group of two terms.
4. Factor the greatest common factor from the results of step 3.

12. \(x^2 - 6x + 5x - 30\)
   \[x(x-6) + 5(x-6)\]
   \[(x-6)(x+5)\]

18. \(a^2 + a + 3a + 3\)
   \[a(a+1) + 3(a+1)\]
   \[(a+1)(a+3)\]

20. \(5x^2 + 30x - 5x - 18\)
   \[5x(x+6) - 3(x+6)\]
   \[(x+6)(5x-3)\]

25. \(x^2 + 9x - x - 9\)
   \[x(x+9) - 1(x+9)\]
   \[(x+9)(x-1)\]
24. \(12x^2 + 42x - 10x - 35\)

28. \(10e^2 + 25e - 6e - 15\)

52. \(3z^4 - 3z^3 - 7z^2 + 7z^2\)

42. \(y^3 - yb + ya - ab\)
32. $3x^2 - 18xy + 4xy - 24y^2$

56. $18x^2 + 27xy + 12xy + 18y^2$