7.1 The Cartesian Coordinate System and Linear Equations in Two Variables

1 Plot points in the Cartesian coordinate system.

2 Determine whether an ordered pair is a solution to a linear equation.

Plot Points in the Cartesian Coordinate System

Many algebraic relationships are easier to understand if we can see a picture of them. A graph shows the relationship between two variables in an equation. In this chapter we discuss several procedures that can be used to draw graphs using the Cartesian (or rectangular) coordinate system. The Cartesian coordinate system is named for its developer, the French mathematician and philosopher René Descartes (1596–1650).

The Cartesian coordinate system provides a means of locating and identifying points just as the coordinates on a map help us find cities and other locations. Consider the map of the Great Smoky Mountains (see Fig. 7.1). Can you find Cades Cove on the map? If we tell you that it is in grid A3, you can probably find it much more quickly and easily.

The Cartesian coordinate system is a grid system, like that of a map, except that it is formed by two axes (or number lines) drawn perpendicular to each other. The two intersecting axes form four quadrants, numbered I through IV in Figure 7.2.

The horizontal axis is called the x-axis. The vertical axis is called the y-axis. The point of intersection of the two axes is called the origin. At the origin the value of x is 0 and the value of y is 0. Starting from the origin and moving to the right along the x-axis, the numbers increase (Fig. 7.3). Starting from the origin and moving to the left, the numbers decrease. Starting from the origin and moving up the y-axis, the numbers increase. Starting from the origin and moving down, the numbers decrease.

\((x, y)\)
To locate a point, it is necessary to know both the value of $x$ and the value of $y$, or the coordinates of the point. When the $x$- and $y$-coordinates of a point are placed in parentheses, with the $x$-coordinate listed first, we have an ordered pair. In the ordered pair $(3, 5)$ the $x$-coordinate is 3 and the $y$-coordinate is 5. The point corresponding to the ordered pair $(3, 5)$ is plotted in Figure 7.4. The phrase “the point corresponding to the ordered pair $(3, 5)$” is often abbreviated “the point $(3, 5)$.” For example, if we write “the point $(-1, 2)$,” it means “the point corresponding to the ordered pair $(-1, 2)$.”

**EXAMPLE 1** Plot (or mark) each point on the same axes.

- **a)** $A(5, 3)$
- **b)** $B(2, 4)$
- **c)** $C(-3, 1)$
- **d)** $D(4, 0)$
- **e)** $E(-2, -5)$
- **f)** $F(0, -3)$
- **g)** $G(0, 2)$
- **h)** $H\left(6, -\frac{9}{2}\right)$
- **i)** $I\left(-\frac{3}{2}, -\frac{5}{2}\right)$

**28.** List the ordered pairs corresponding to each point.

- **A** $(8, 5)$
- **B** $(4, 0)$
- **C** $(6, -15)$
- **D** $(-4, 20)$
- **E** $(0, -20)$
- **F** $(-6, -5)$
- **G** $(-5, 10)$

**Ordered Pairs** $(x, y)$
2 Determine Whether an Ordered Pair Is a Solution to a Linear Equation

In Section 7.2 we will learn to graph linear equations in two variables. Below we explain how to identify a linear equation in two variables.

**Linear Equations in Two Variables**

A linear equation in two variables is an equation that can be put in the form

\[ ax + by = c \]

where \( a, b, \) and \( c \) are real numbers.

The graphs of equations of the form \( ax + by = c \) are straight lines. For this reason such equations are called linear. Linear equations may be written in various forms, as we will show later. A linear equation in the form \( ax + by = c \) is said to be in standard form.

**Examples of Linear Equations**

\[
\begin{align*}
4x - 3y & = 12 \\
y & = 5x + 3 \\
x - 3y + 4 & = 0
\end{align*}
\]

How many possible solutions does the equation \( y = x + 1 \) have? The equation \( y = x + 1 \) has an unlimited or infinite number of possible solutions. Since it is not possible to list all the specific solutions, the solutions are illustrated with a graph.

**Graph of an Equation**

A graph of an equation in two variables is an illustration of a set of points whose coordinates satisfy the equation.

*Figure 7.7a* shows the points \((2, 3), (-3, -2),\) and \((\frac{1}{3}, \frac{2}{3})\) plotted in the Cartesian coordinate system. *Figure 7.7b* shows a straight line drawn through the three points. Arrowheads are placed at the ends of the line to show that the line continues in both directions. Every point on this line will satisfy the equation \( y = x + 1 \), so this graph illustrates all the solutions of \( y = x + 1 \). The ordered pair \((1, 2)\), which is on the line, also satisfies the equation.

In *Figure 7.7b*, what do you notice about the points \((2, 3), (1, 2), (\frac{1}{3}, \frac{2}{3}),\) and \((-3, -2)\)? You probably noticed that they are in a straight line. A set of points that are in a straight line are said to be collinear. In Section 7.2 when you graph linear equations by plotting points, the points you plot should all be collinear.

34. \( A(1, -2), B(0, -5), C(4, 1), D(-1, -8), E\left(\frac{1}{2}, -\frac{7}{2}\right) \)
**Helpful Hint**

Only two points are needed to graph a linear equation because the graph of every linear equation is a straight line. However, if you graph a linear equation using only two points and you have made an error in determining or plotting one of those points, your graph will be wrong and you will not know it. In Figures 7.9(a) and (b) we plot only two points to show that if only one of the two points plotted is incorrect, the graph will be wrong. In both Figures 7.9(a) and (b) we use the ordered pair $(-2,-2)$. However, in Figure 7.9(a) the second point is $(1,2)$, while in Figure 7.9(b) the second point is $(2,1)$. Notice how the two graphs differ.

If you use at least three points to plot your graph, as in Figure 7.7b on page 426, and they appear to be collinear, you probably have not made a mistake.

**FIGURE 7.9**

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In Exercises 37–42, **a)** determine which of the four ordered pairs does not satisfy the given equation. **b)** Plot all the points that satisfy the equation on the same axes and draw a straight line through the points.

38. $2x + y = -4$,

- **a)** $(-2,0)$
- **b)** $(2,3)$
- **c)** $(0,-4)$
- **d)** $(-1,-2)$

**a)**

- $2x+y=-4$
- $(2,0)$ **Yes**
- $2(-2)+0=-4$
- $-4=-4$

**b)**

- $(2,3)$ **No**!
- $2(2)+3=-4$
- $4+3=-4$
- $-4=-4$

**c)**

- $(0,-4)$ **Yes**!
- $2(0)-y=-4$
- $-4=-4$
Consider the linear equation $y = 3x - 4$. In Exercises 43–46, find the value of $y$ that makes the given ordered pair a solution to the equation.

43. $(2, y)$
   
   \[
   y = 3(2) - 4 \\
   y = 6 - 4 \\
   y = 2 \\
   (2, 2)
   \]

44. $(-1, y)$
   
   \[
   y = 3(-1) - 4 \\
   y = -3 - 4 \\
   y = -7 \\
   (-1, -7)
   \]

45. $(0, y)$
   
   \[
   y = 3(0) - 4 \\
   y = 0 - 4 \\
   y = -4 \\
   (0, -4)
   \]

46. $(3, y)$
   
   \[
   y = 3(3) - 4 \\
   y = 9 - 4 \\
   y = 5 \\
   (3, 5)
   \]
Consider the linear equation $2x + 3y = 12$. In Exercises 47–50, find the value of $x$ that makes the given ordered pair a solution to the equation.

47. $(x, 2)$

$$2x + 3(2) = 12$$
$$2x + 6 = 12$$
$$2x = 6$$
$$x = 3$$

48. $(x, 4)$

$$2x + 3(4) = 12$$
$$2x + 12 = 12$$
$$2x = 0$$
$$x = 0$$

49. \( \left( x, \frac{11}{3} \right) \)

50. \( \left( x, \frac{22}{3} \right) \)

\((-5, \frac{11}{3})\)