\[
\frac{-[3 - (8 - 16)^2]}{[15 \div 5 + 5]^2 + 3^2} = \frac{61}{73}
\]
\[ \{8 - 5[6 - (15 ÷ 3)]^2\}^2 \]

\[ \{8 - 5[6 - 5]^2\}^2 \]

\[ \{8 - 5[1]^2\}^2 \]

\[ \{8 - 5\cdot1\}^2 \]

\[ \{8 - 5\}^2 \]

\[ (3)^2 = 9 \]
1.10 Properties of the Real Number System

Here, we introduce various properties of the real number system.

1. Learn the Commutative Property

The **commutative property of addition** states that the order in which any two real numbers are added does not matter.

<table>
<thead>
<tr>
<th>Commutative Property of Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>If (a) and (b) represent any two real numbers, then</td>
</tr>
<tr>
<td>(a + b = b + a)</td>
</tr>
</tbody>
</table>

Notice that the commutative property involves a change in *order*. For example,

\[
\begin{align*}
4 + 3 &= 3 + 4 \\
7 &= 7
\end{align*}
\]

The **commutative property of multiplication** states that the order in which any two real numbers are multiplied does not matter.

<table>
<thead>
<tr>
<th>Commutative Property of Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>If (a) and (b) represent any two real numbers, then</td>
</tr>
<tr>
<td>(a \cdot b = b \cdot a)</td>
</tr>
</tbody>
</table>

For example,

\[
\begin{align*}
6 \cdot 3 &= 3 \cdot 6 \\
18 &= 18
\end{align*}
\]

*The commutative property does not hold for subtraction or division.* For example,

\(4 - 6 \neq 6 - 4\) and \(6 \div 3 \neq 3 \div 6\).
2 Learn the Associative Property

The **associative property of addition** states that, in the addition of three or more numbers, parentheses may be placed around any two adjacent numbers without changing the results.

### Associative Property of Addition

If $a$, $b$, and $c$ represent any three real numbers, then

$$(a + b) + c = a + (b + c)$$

Notice that the associative property involves a change of *grouping*. For example,

$$(3 + 4) + 5 = 3 + (4 + 5)$$

$$7 + 5 = 3 + 9$$

$$12 = 12$$

In this example, the 3 and 4 are grouped together on the left, and the 4 and 5 are grouped together on the right.

The **associative property of multiplication** states that, in the multiplication of three or more numbers, parentheses may be placed around any two adjacent numbers without changing the results.

### Associative Property of Multiplication

If $a$, $b$, and $c$ represent any three real numbers, then

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

For example,

$$(6 \cdot 2) \cdot 4 = 6 \cdot (2 \cdot 4)$$

$$12 \cdot 4 = 6 \cdot 8$$
Distributive Property

If $a$, $b$, and $c$ represent any three real numbers, then

$$a(b + c) = ab + ac$$

For example, if we let $a = 2$, $b = 3$, and $c = 4$, then

$$2(3 + 4) = (2 \cdot 3) + (2 \cdot 4)$$
$$2 \cdot 7 = 6 + 8$$
$$14 = 14$$

Therefore, we may either add first and then multiply, or multiply first and then add. Another example of the distributive property is

$$2(x + 3) = 2 \cdot x + 2 \cdot 3 = 2x + 6$$

The distributive property can be expanded in the following manner:

$$a(b + c + d + \cdots + n) = ab + ac + ad + \cdots + an$$

For example, $3(x + y + 5) = 3x + 3y + 15$.

**Helpful Hint**

The commutative property changes order.
The associative property changes grouping.
The distributive property involves two operations, usually multiplication and addition.
EXAMPLE 1 Name each property illustrated.

a) $4 + (-2) = -2 + 4 \quad \text{Commutative Prop.}

b) $5(r + s) = 5 \cdot r + 5 \cdot s = 5r + 5s

c) $x \cdot y = y \cdot x \quad \text{Commutative Prop. Mult.}

d) $(-12 + 3) + 4 = -12 + (3 + 4) \quad \text{Associative Prop. Add.}$
4 Learn the Identity Properties

Now we will discuss the identity properties. When the number 0 is added to any real number, the real number is unchanged. For example, $5 + 0 = 5$ and $0 + 5 = 5$. For this reason we call 0 the identity element of addition or additive identity. When any real number is multiplied by 1, the real number is unchanged. For example, $7 \cdot 1 = 7$ and $1 \cdot 7 = 7$. For this reason we call 1 the identity element of multiplication or multiplicative identity.

**Identity Properties**

If $a$ represents any real number, then

$$a + 0 = a \quad \text{and} \quad 0 + a = a \quad \text{Identity property of addition}$$

and

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a \quad \text{Identity property of multiplication}$$

We often use the identity properties without realizing we are using them. For example, when we reduce $\frac{15}{50}$, we may do the following:

$$\frac{15}{50} = \frac{3 \cdot 5}{10 \cdot 5} = \frac{3}{10} \cdot \frac{5}{5} = \frac{3}{10} \cdot 1 = \frac{3}{10}$$

When we showed that $\frac{3}{10} \cdot 1 = \frac{3}{10}$, we used the identity property of multiplication.


5 Learn the Inverse Properties

The last properties we will discuss in this chapter are the inverse properties. Numbers like 3 and \(-3\) are opposites or additive inverses because \(3 + (-3) = 0\) and \(-3 + 3 = 0\). Any two numbers whose sum is 0 are called additive inverses of each other. In general, for any real number \(a\) its additive inverse is \(-a\).

Numbers like 4 and \(\frac{1}{4}\) are reciprocals or multiplicative inverses because \(4 \cdot \frac{1}{4} = 1\) and \(\frac{1}{4} \cdot 4 = 1\). Any two numbers whose product is 1 are called multiplicative inverses of each other. In general, for any real number \(a\), its multiplicative inverse is \(\frac{1}{a}\). The inverse properties are summarized below.

<table>
<thead>
<tr>
<th>Inverse Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>If (a) represents any real number, then</td>
</tr>
<tr>
<td>(a + (-a) = 0) and (-a + a = 0) <strong>Inverse property of addition</strong></td>
</tr>
<tr>
<td>and</td>
</tr>
<tr>
<td>(a \cdot \frac{1}{a} = 1) and (\frac{1}{a} \cdot a = 1 (a \neq 0)) <strong>Inverse property of multiplication</strong></td>
</tr>
</tbody>
</table>

We often use the inverse properties without realizing we are using them. For example, to evaluate the expression \(6x + 2\) when \(x = \frac{1}{6}\), we may do the following:

\[
6x + 2 = 6\left(\frac{1}{6}\right) + 2 = 1 + 2 = 3
\]

When we multiplied \(6\left(\frac{1}{6}\right)\) and replaced it with 1, we used the inverse property of multiplication. We will be using both the identity and inverse properties throughout the book, although we may not specifically refer to them by name.
EXAMPLE 2  Name each property illustrated.

a) \(2(x + 6) = (2 \cdot x) + (2 \cdot 6) = 2x + 12\)  Distributive Prop.

b) \(3x \cdot 1 = 3x\)

c) \((3 \cdot 6) \cdot 5 = 3 \cdot (6 \cdot 5)\)  Associative Prop. Mult.

d) \(\frac{1}{y} \cdot y = 1\)

e) \(2a + (-2a) = 0\)  Additive Inverse

f) \(3y + 0 = 3y\)  Additive Identity
EXAMPLE 3 In parts a)–f), the name of a property is given followed by part of an equation. Complete the equation, to the right of the equals sign, to illustrate the given property.

a) Associative property of multiplication
\[(5 \cdot 4) \cdot 7 = 5 \cdot (4 \cdot 7)\]

b) Inverse property of addition
\[3c + (-3c) = 0\]

c) Identity property of multiplication
\[6y \cdot 1 = 6y\]

d) Distributive property
\[3(x + 5) = 3x + 15\]

e) Identity property of addition
\[2a + 0 = 2a\]

f) Inverse property of multiplication
\[b \cdot \frac{1}{b} = 1\]

\[\frac{1}{b} \cdot b = 1\]
Practice the Skills

In Exercises 11–22, for the given expression, determine a) the additive inverse, and b) the multiplicative inverse.

11. \(6 + (-6) = 0\)
   \(\frac{6}{1} \cdot \frac{-1}{-6} = 1\)

12. \(\frac{5 + (-5)}{\frac{2}{1} \cdot \frac{1}{5}} = 1\)

13. \(-3\)

14. \(-7\)

15. \(x\)

16. \(z\)

17. \(1.6 \div (1.6) = 1\)
   \(\frac{1.6}{1} \cdot \frac{1}{1.6} = 1\)
   \(\frac{1}{1.25} = 0.8\)

18. \(-0.125\)

19. \(\frac{1}{5}\)

20. \(\frac{-125}{1000} \div \frac{125}{1000} = 1\)
   \(\frac{1}{-1.25} = -0.8\)

21. \(\frac{1.25}{1} \cdot \frac{1}{1.25} = 1\)

22. \(\frac{1}{1.25} \div \frac{1}{5} = \frac{1}{1.25} \cdot \frac{5}{1} = \frac{5}{1} = 5\)
Name each property illustrated.

24. \(3 + y = y + 3\)  \(\quad\) Comm. Add.
26. \(1(x + 3) = (1)(x) + (1)(3) = x + 3\)  \(\quad\) Distributive Prop.
28. \(-4x + 4x = 0\)  \(\quad\) Additive Invers.
30. \(2(x + 4) = 2x + 8\)  \(\quad\) Distributive Prop.
32. \(3 + (4 + t) = (3 + 4) + t\)  \(\quad\) Associative of Add.
34. \(0 + 3y = 3y\)  \(\quad\) Additive Identity
36. \(x \cdot y = y \cdot x\)  \(\quad\) Comm. of Mult.
In Exercises 37–58, the name of a property is given followed by part of an equation. Complete the equation, to the right of the equals sign, to illustrate the given property.

38. inverse property of addition
   \((-7a) + 7a = \) \(0\)

40. associative property of addition
   \(-5 + (6 + 8) = \)

42. distributive property
   \(4(\chi + 3) = \)

44. identity property of multiplication
   \((1)(-\frac{1}{3}b) = \)

46. associative property of multiplication
   \(-9 \cdot (3 \cdot 8) = \)

48. commutative property of multiplication
   

   \((\chi + 2)3 = 3(\chi + 2)\)

50. commutative property of addition
   \(3(\chi + y) = 3(y + \chi)\)

52. associative property of multiplication
   \((3\chi)y = 3(\chi y)\)

54. distributive property
   \(3(\chi + y + 2) = \)

56. identity property of addition
   \(0 + 2\chi = \)

58. inverse property of multiplication
   \(\left(\frac{1}{2}\right)\left(\frac{2}{\chi}\right) = \)