Section 2.1  #30  MML

\[ \frac{1}{2}(x + 9) + \frac{1}{3}(3x + 12) \]

\[ \frac{1}{2} \cdot \frac{7}{4} = \frac{9}{2} \]

\[ \frac{1}{2}x + \frac{9}{2} + 1x + 4 \]

\[ \frac{3}{2}x + \frac{9}{2} \]

\[ 1 \frac{1}{2}x + 8 \frac{1}{2} \]

\[ \frac{1}{3} \cdot \frac{3}{1} = \frac{3}{3} = 1 \]

\[ \frac{1}{3} \cdot \frac{4}{1} = 4 \]

\[ 3 \frac{1}{2}x \]
\[ \frac{6}{x} + \frac{1}{6(7x - 12)} \]

\[ \frac{6}{x} + \frac{1}{6} \cdot \frac{2}{7x - 12} \]

\[ \frac{6}{x} \cdot \frac{7x - 12}{7x - 12} + \frac{2}{7} \cdot \frac{6}{7x - 12} \]

\[ \frac{42}{7x} x + \frac{12}{7x} x - \frac{2}{7x} \]

\[ \frac{54}{7x} x - \frac{2}{7x} \]

\[ \frac{54}{7x} x - 2 \]

\[ \frac{1}{7x} \cdot \frac{2}{1} \cdot 2 \]
Simplify.

\[
0.3 - (y + 3) + 0.7 - 6
\]

\[
0.3 - y - 3 + 0.7 - 6
\]

\[
-\frac{3}{\text{.7}} - \frac{-3}{\text{-6}} - \frac{1}{\text{-9}}
\]
2.2 The Addition Property of Equality

1 Identify Linear Equations

A statement that shows two algebraic expressions are equal is called an equation. For example, $4x + 3 = 2x - 4$ is an equation. In this chapter, we learn to solve linear equations in one variable.

<table>
<thead>
<tr>
<th>Linear Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A linear equation in one variable is an equation that can be written in the form $ax + b = c$ where $a$, $b$, and $c$ are real numbers and $a \neq 0$.</td>
</tr>
</tbody>
</table>

Examples of Linear Equations
- $x + 4 = 7$
- $2x - 4 = 6$

2 Check Solutions to Equations

The solution to an equation is the number or numbers that when substituted for the variable or variables make the equation a true statement. For example, the solution to $x + 4 = 7$ is 3. We will shortly learn how to find the solution to an equation, or to solve an equation. But before we do this we will learn how to check the solution to an equation.

The solution to an equation may be checked by substituting the value that is believed to be the solution for the variable in the original equation. If the substitution results in a true statement, your solution is correct. If the substitution results in a false statement, then either your solution or your check is incorrect, and you need to go back and find your error. Try to check all your solutions. Checking the solutions will improve your arithmetic and algebra skills.

When we show the check of a solution we shall use the notation, $\checkmark$. This notation is used when we are questioning whether a statement is true. For example, if we use

$$2 + 3 \overset{?}{=} 2(3) - 1$$

we are asking “Does $2 + 3 = 2(3) - 1$?”

To check whether 3 is the solution to $x + 4 = 7$, we substitute 3 for each $x$ in the equation.

Check:

$$x = 3$$
$$x + 4 = 7$$
$$3 + 4 \overset{?}{=} 7$$
$$7 = 7 \quad \text{True}$$
14. Is \( x = -6 \) a solution of \( 2x + 1 = x - 5 \)?

\[
\begin{align*}
2x + 1 &= x - 5 \\
2(-6) + 1 &= -6 - 5 \\
-12 + 1 &= -11 \\
-11 &= -11 \\
\text{True} \\
\end{align*}
\]

\( x = -6 \) is a solution of this equation.

18. Is \( k = -2 \) a solution of \( 5k - 6(k - 1) = 8 \)?

\[
\begin{align*}
5(-2) - 6(-2) &= 8 \\
-10 + 12 &= 8 \\
2 &= 8 \
\end{align*}
\]

True

24. Is \( h = 3 \) a solution of \(- (h - 5) - (h - 6) = 3h - 4\)?

\[
\begin{align*}
- (h - 5) - (h - 6) &= 3h - 4 \\
- (3 - 5) - (3 - 6) &= 3(3) - 4 \\
10 - 9 &= 9 - 4 \\
2 + 3 &= 5 \
\end{align*}
\]

True
Identify Equivalent Equations

Now that we know how to check a solution to an equation, we will discuss solving equations. Complete procedures for solving equations will be given shortly. For now, you need to understand that to solve an equation, it is necessary to get the variable alone on one side of the equal sign. That is, we want to get an equation of the form \( x = \) some number (or \( 1x = \) some number). When we get an equation in this form, we say that we isolate the variable. To isolate the variable, we make use of two properties: the addition and multiplication properties of equality. Look first at Figure 2.1.

Think of an equation as a balanced statement whose left side is balanced by its right side. When solving an equation, we must make sure that the equation remains balanced at all times. That is, both sides must always remain equal. We ensure that an equation always remains equal by doing the same thing to both sides of the equation. For example, if we add a number to the left side of the equation, we must add exactly the same number to the right side. If we multiply the right side of the equation by some number, we must multiply the left side by the same number.

When we add the same number to both sides of an equation or multiply both sides of an equation by the same nonzero number, we do not change the solution to the equation; just the form of the equation. Two or more equations with the same solution are called equivalent equations. The equations \( 2x - 4 = 2 \), \( 2x = 6 \), and \( x = 3 \) are equivalent, since the solution to each is 3.

Check: \( x = 3 \)

\[
\begin{align*}
2x - 4 &= 2 & 2x &= 6 & x &= 3 \\
2(3) - 4 &= 2 & 2(3) &= 6 & 3 &= 3 & \text{True} \\
6 - 4 &= 2 & 6 &= 6 & \text{True} \\
2 &= 2 & \text{True}
\end{align*}
\]

When solving an equation, we use the addition and multiplication properties to express a given equation as simpler equivalent equations until we obtain the solution.
4 Use the Addition Property to Solve Equations

Now we are ready to define the addition property of equality.

**Addition Property of Equality**

If $a = b$, then $a + c = b + c$ for any real numbers $a$, $b$, and $c$.

This property means that the same number can be added to both sides of an equation without changing the solution. The addition property is used to solve equations of the form $x + a = b$. To isolate the variable $x$ in equations of this form, add the opposite or additive inverse of $a$, $-a$, to both sides of the equation.

To isolate the variable when solving equations of the form $x + a = b$, we use the addition property to eliminate the number on the same side of the equal sign as the variable. Study the following examples carefully.

<table>
<thead>
<tr>
<th>Equation</th>
<th>To Solve, Use the Addition Property to Eliminate the Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - 4 = -3$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$x + 5 = 9$</td>
<td>$5$</td>
</tr>
<tr>
<td>$-3 = k + 7$</td>
<td>$7$</td>
</tr>
<tr>
<td>$-5 = x - 4$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$-6.25 = y + 12.78$</td>
<td>$12.78$</td>
</tr>
</tbody>
</table>

30. $x - 16 = -12$

32. $x + 8 = 17$

28. $-5 = x + 4$

38. $-4 = x - 3$

68. $-5.62 = y + 11.39$
EXERCISE SET 2.2

Concept/Writing Exercises

1. What is an equation?
2. a) What is meant by the “solution to an equation”?
   b) What does it mean to “solve an equation”?
3. Explain how the solution to an equation may be checked.
4. Explain the addition property of equality.
5. What are equivalent equations?
6. To solve an equation we “isolate the variable.”
   a) Explain what this means.
   b) Explain how to isolate the variable in the equations discussed in this section.
7. When solving the equation 6 = x + 2, would you subtract 6 from both sides of the equation or subtract 2 from both sides of the equation? Explain.
8. When solving the equation x - 4 = 6, would you add 4 to both sides of the equation or subtract 6 from both sides of the equation? Explain.
9. Give an example of a linear equation in one variable.
10. Explain why the addition property allows us to subtract the same quantity from both sides of an equation.
11. Explain why the following three equations are equivalent.
    \[ 2x + 3 = 5, \quad 2x = 2, \quad x = 1 \]
12. To solve the equation x - □ = △ for x, do we add □ to both sides of the equation or do we subtract △ from both sides of the equation? Explain.