2.3 The Multiplication Property of Equality

Identify Reciprocals

In Section 1.10, we introduced the reciprocal (or multiplicative inverse) of a number. Recall that two numbers are reciprocals of each other when their product is 1. Some examples of numbers and their reciprocals follow.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reciprocal</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1/2</td>
<td>(2)(1/2) = 1</td>
</tr>
<tr>
<td>-3/5</td>
<td>-5/3</td>
<td>(-3/5)(-5/3) = 1</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>(-1)(-1) = 1</td>
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</tbody>
</table>

The reciprocal of a positive number is a positive number and the reciprocal of a negative number is a negative number. Note that 0 has no reciprocal. Why?

In general, if \( a \) represents any nonzero number, its reciprocal is \( \frac{1}{a} \). For example, the reciprocal of 3 is \( \frac{1}{3} \) and the reciprocal of \(-2\) is \( \frac{1}{-2} \) or \( -\frac{1}{2} \). The reciprocal of \(-\frac{3}{5}\) is \( \frac{1}{-\frac{3}{5}} \), which can be written as \( 1 \div \left(-\frac{3}{5}\right) \). Simplifying, we get \( \left(\frac{1}{1}\right)\left(-\frac{5}{3}\right) = -\frac{5}{3} \).

Thus, the reciprocal of \(-\frac{3}{5}\) is \(-\frac{5}{3}\). 
2 Use the Multiplication Property to Solve Equations

In Section 2.2, we used the addition property of equality to solve equations of the form \( x + a = b \), where \( a \) and \( b \) represent real numbers. In this section, we use the multiplication property of equality to solve equations of the form \( ax = b \), where \( a \) and \( b \) represent real numbers.

It is important that you recognize the difference between equations like \( x + 2 = 8 \) and \( 2x = 8 \). In \( x + 2 = 8 \), the 2 is a term that is being added to \( x \), so we use the addition property to solve the equation. In \( 2x = 8 \), the 2 is a factor of \( 2x \). The 2 is the coefficient multiplying the \( x \), so we use the multiplication property to solve the equation. The multiplication property of equality is used to solve linear equations where the coefficient of the \( x \)-term is a number other than 1.

Now we present the multiplication property of equality.

**Multiplication Property of Equality**

If \( a = b \), then \( a \cdot c = b \cdot c \) for any real numbers \( a \), \( b \), and \( c \).

The multiplication property means that both sides of an equation can be multiplied by the same nonzero number without changing the solution. The multiplication property **can be used to solve equations of the form** \( ax = b \). We can isolate the variable in equations of this form by multiplying both sides of the equation by the reciprocal of \( a \), which is \( \frac{1}{a} \). Doing so makes the numerical coefficient of the variable, \( x \), become 1, which can be omitted when we write the variable.

10. \( 5x = 50 \)

\[
\frac{5x}{5} = \frac{50}{5} \quad \Rightarrow \quad x = 10
\]

12. \( \frac{y}{3} = 3 \)

\[
\frac{\frac{y}{3}}{\frac{1}{3}} = \frac{3 \cdot \frac{y}{3}}{3 \cdot \frac{1}{3}} \quad \Rightarrow \quad y = 15
\]

50. \( \frac{2}{7}x = 7 \)

\[
\frac{\frac{2}{7}x}{\frac{2}{7}} = \frac{7}{\frac{2}{7}} \quad \Rightarrow \quad x = \frac{49}{2}
\]

In Example 1, we multiplied both sides of the equation \( 9x = 63 \) by \( \frac{1}{9} \) to isolate the variable. We could have also isolated the variable by dividing both sides of the equation by 9, as follows:

\[
\frac{9x}{9} = \frac{63}{9} \quad \Rightarrow \quad \frac{1}{9}x = \frac{7}{1} \quad \text{Divide both sides by 9.}
\]

\[
x = 7
\]

We can do this because dividing by 9 is equivalent to multiplying by \( \frac{1}{9} \). **Since division can be defined in terms of multiplication** \( \left( \frac{a}{b} \right. \text{ means } a \cdot \frac{1}{b} \). **the multiplication property also allows us to divide both sides of an equation by the same nonzero number.** This process is illustrated in Examples 4 through 6.
58. \(-9 = \frac{-5}{3} n\)

\[-9 = -\frac{5}{3} n\]

\[\left(\frac{-2}{3}\right) \left(\frac{-9}{7}\right) = \left(\frac{-2}{3}\right) \left(\frac{-1}{3}\right)\]

\[\frac{22}{\frac{8}{3}} = 1 n\]

\[\frac{22}{\frac{8}{3}} = n\]

40. \(-2b = -\frac{4}{5}\)

\[-2b = \frac{-4}{5}\]

\[-\frac{2}{-2} = \frac{-4}{5} \cdot \frac{5}{-2}\]

\[-\frac{2}{2} = \frac{-4}{5} \cdot \frac{5}{-2}\]

\[-\frac{2}{2} = \frac{2}{5}\]

\[b = \frac{2}{5}\]

\[\left(\frac{-1}{2}\right) - \frac{2}{1} = \frac{b - \left(\frac{1}{2}\right)}{5}\]

\[b = \frac{2}{5}\]

\[\frac{2}{2} \cdot \frac{2}{3} x = \frac{3}{2} \cdot \frac{12}{5}\]

\[x = 18\]

22. \(16 = -4y\)

\[\frac{16}{-4} = \frac{-4y}{-4}\]

\[y = 4\]

36. \(-3.88 = 1.94y\)

\[\frac{-3.88}{1.94} = \frac{1.94y}{1.94}\]

\[y = -2\]

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3 Solve Equations of the Form \(-x = a\)

When solving an equation, we may obtain an equation like \(-x = 7\). This is not a solution since \(-x = 7\) means \(-1x = 7\). The solution to an equation is of the form \(x = \) some number. When an equation is of the form \(-x = 7\), we can solve for \(x\) by multiplying both sides of the equation by \(-1\), as illustrated in the following example.

**EXAMPLE 9** Solve the equation \(-x = 7\).

**Solution** \(-x = 7\) means that \(-1x = 7\). We are solving for \(x\), not \(-x\). We can multiply both sides of the equation by \(-1\) to isolate \(x\) on the left side of the equation.

\[
\begin{align*}
-x &= 7 \\
-1x &= 7 \\
(-1)(-1x) &= (-1)(7) \\
1x &= -7 \\
x &= -7
\end{align*}
\]

**Check:**

\[
\begin{align*}
-x &= 7 \\
-(-7) &= 7 \\
7 &= 7 & \text{True}
\end{align*}
\]

Thus, the solution is \(-7\).

24. \(-x = 9\)  
26. \(-x = -15\)  
62. \(-9x = -45\)

64. \(\frac{1}{3}x = 15\)

\[
\begin{align*}
\frac{1}{3}x &= \frac{15}{3} \\
x &= 45
\end{align*}
\]

\[
\begin{align*}
\frac{15}{1} \div \frac{1}{3} &= \frac{15}{1} \times \frac{3}{1} \\
\frac{1}{3}x &= \frac{15}{1} \div \frac{3}{1} \\
x &= 45
\end{align*}
\]
Concept/Writing Exercises

1. Explain the multiplication property of equality.

2. Explain why the multiplication property allows us to divide both sides of an equation by a nonzero quantity.

3. a) If \(-x = a\), where \(a\) represents any real number, what does \(x\) equal?
   b) If \(-x = 5\), what is \(x\)?
   c) If \(-x = -5\), what is \(x\)?

4. When solving the equation \(-2x = 5\), would you divide both sides of the equation by \(-2\) or by \(5\)? Explain.

5. When solving the equation \(3x = 5\), would you divide both sides of the equation by \(3\) or by \(5\)? Explain.

6. When solving the equation \(4 = \frac{x}{3}\), what would you do to isolate the variable? Explain.

7. When solving the equation \(x = 3\), what would you do to isolate the variable? Explain.

8. When solving the equation \(ax = b\) for \(x\), would you divide both sides of the equation by \(a\) or \(b\)? Explain.