1.8 Multiplication and Division of Real Numbers

1. Multiply numbers.
2. Divide numbers.
3. Remove negative signs from denominators.
4. Evaluate divisions involving 0.

1. Multiply Numbers

The following rules are used in determining the sign of the product when two numbers are multiplied.

<table>
<thead>
<tr>
<th>The Sign of the Product of Two Real Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The product of two numbers with like signs is a <strong>positive</strong> number.</td>
</tr>
<tr>
<td>2. The product of two numbers with unlike signs is a <strong>negative</strong> number.</td>
</tr>
</tbody>
</table>

The product of two positive numbers or two negative numbers will be a positive number. The product of a positive number and a negative number will be a negative number.

**EXAMPLE 1** Evaluate.

\[
\begin{align*}
\text{a) } 4(-5) & -20 \\
\text{b) } (-6)(7) & -42 \\
\text{c) } (-9)(-3) & 27
\end{align*}
\]

**EXAMPLE 2** Evaluate.

\[
\begin{align*}
\text{a) } (-8)(5) & -40 \\
\text{b) } (-4)(-8) & 32 \\
\text{c) } 0(6) & 0 \\
\text{d) } 0(-2) & 0
\end{align*}
\]
EXAMPLE 4 Evaluate.

a) \((-5)(-3)(1)(-4) = -60\)

b) \((-2)(-4)(-1)(3)(-4) = +96\)

\((1)(-2) = -2\)
\((1)(-2)(-3) = -6\)
\((1)(-2)(-3)(-4) = 24\)
\((1)(-2)(-1)(-4)(-5) = -120\)

Event of Negative Signs
Answer is Positive
Odd # of Negative Signs
Answer is Negative
2. **Divide Numbers**

The rules for dividing numbers are very similar to those used in multiplying numbers.

<table>
<thead>
<tr>
<th>The Sign of the Quotient of Two Real Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The quotient of two numbers with <em>like</em> signs is a <strong>positive</strong> number.</td>
</tr>
<tr>
<td>2. The quotient of two numbers with <em>unlike</em> signs is a <strong>negative</strong> number.</td>
</tr>
</tbody>
</table>

**EXAMPLE 5** Evaluate.

\[
\begin{align*}
a) \quad \frac{10}{-5} & = -2 \\
b) \quad \frac{-45}{5} & = -9 \\
c) \quad \frac{-36}{-6} & = 6
\end{align*}
\]

**EXAMPLE 6** Evaluate.

\[
\begin{align*}
a) \quad -16 \div (-2) & = 8 \\
b) \quad \frac{-2}{3} \div \frac{-5}{7} & = \frac{14}{15} \\
c) \quad \frac{2}{3} \times \frac{2}{3} & = \end{align*}
\]
EXAMPLE 7 Evaluate, rounding your answer to the nearest hundredth when appropriate.

\[ a) -18.86 \div 4.1 \quad b) \frac{-27.2}{-2.6} \]

**Helpful Hint**

For multiplication and division of two real numbers:

\[
\begin{align*}
(+) (+) &= + \\
(-) (-) &= + \\
(+) (-) &= - \\
(-) (+) &= -
\end{align*}
\]

Like signs give positive products and quotients.

Unlike signs give negative products and quotients.
The Quotient of a Positive Number and a Negative Number

If \( a \) and \( b \) represent any real numbers, \( b \neq 0 \), then

\[
\frac{a}{-b} = -\frac{a}{b} = \frac{-a}{b}
\]

\( \frac{5}{-7} \) should be written as either \( -\frac{5}{7} \) or \( \frac{5}{-7} \).

\[
- \frac{5}{7} = -\frac{5}{7} = \frac{5}{-7}
\]

\[
\frac{2}{3} - \frac{3}{4} = \frac{2}{3} + -\frac{3}{4} = \frac{8}{12} + -\frac{9}{12} = \frac{-1}{12}
\]
4 Evaluate Divisions Involving 0

What is \( \frac{0}{1} \) equal to? Note that \( \frac{6}{3} = 2 \) because \( 3 \cdot 2 = 6 \). We can follow the same procedure to determine the value of \( \frac{0}{1} \). Suppose that \( \frac{0}{1} \) is equal to some number, which we will designate by \( ? \).

If \( \frac{0}{1} = ? \) then \( 1 \cdot ? = 0 \)

Since only \( 1 \cdot 0 = 0 \), the \( ? \) must be 0. Thus, \( \frac{0}{1} = 0 \). Using the same technique, we can show that zero divided by any nonzero number is zero.

Zero Divided by a Nonzero Number

If \( a \) represents any real number except 0, then

\[
0 \div a = \frac{0}{a} = 0
\]

Now what is \( \frac{1}{0} \) equal to?

If \( \frac{1}{0} = ? \) then \( 0 \cdot ? = 1 \)

But since 0 multiplied by any number will be 0, there is no value that can replace \( ? \). We say that \( \frac{1}{0} \) is undefined. Using the same technique, we can show that any real number, except 0, divided by 0 is undefined.

Division by Zero

If \( a \) represents any real number except 0, then

\[
a \div 0 \quad \text{or} \quad \frac{a}{0} \quad \text{is undefined}
\]
EXAMPLE 9 Indicate whether each quotient is 0 or undefined.

a) \( \frac{0}{2} \)  

b) \( \frac{5}{0} \)  

c) \( \frac{0}{-4} \)  

d) \( \frac{-2}{0} \)
20. $6(-2)$
24. $-1(8.7)$
28. $(7)(-8)$
32. $5(-4)(2)$
36. $(2)(-4)(-5)(-1)$
40. $(-6)(6)(4)(-4)$

44. \[\left(\frac{4}{5}\right)\left(\frac{-3}{10}\right)\]
48. \[\left(\frac{-9}{10}\right)\left(\frac{7}{-8}\right)\]
52. \( \frac{-18}{9} \)
56. \( \frac{-15}{-1} \)
60. \( \frac{63}{-7} \)
64. \( \frac{-10}{10} \)

68. Divide 26 by \(-13\).
72. Divide \(-36\) by \(-6\).

76. \( \frac{6}{15} \div \left( \frac{7}{30} \right) \)
80. \( \frac{-16}{3} \div \left( \frac{5}{-9} \right) \)

92. \((6)(1)(-3)(4)\)

94. \( \frac{-2.7}{0} \)

96. \( \frac{0}{1} \)