2.1 Combining Like Terms

1. Identify Terms.

In Section 1.3, we indicated that letters called variables are used to represent numbers. A variable can represent a variety of different numbers.

As was indicated in Chapter 1, an expression (sometimes referred to as an algebraic expression) is a collection of numbers, variables, grouping symbols, and operation symbols.

Examples of Expressions

<table>
<thead>
<tr>
<th>Expression</th>
<th>Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>(x^2 - 6)</td>
<td>(x^2), -6</td>
</tr>
<tr>
<td>4x - 3</td>
<td>4x, -3</td>
</tr>
<tr>
<td>2(x + 5) + 6</td>
<td>2x + 10, 6</td>
</tr>
<tr>
<td>(x + \frac{3}{4})</td>
<td>(x), (\frac{3}{4})</td>
</tr>
</tbody>
</table>

When an algebraic expression consists of several parts, the parts that are added are called the terms of the expression. Consider the expression \(2x - 3y - 5\). The expression can be written as \(2x + (-3y) + (-5)\), and so the expression \(2x - 3y - 5\) has three terms: \(2x\), \(-3y\), and \(-5\). The expression \(3x^2 + 2xy + 5(x + y)\) also has three terms: \(3x^2\), \(2xy\), and \(5(x + y)\).

When listing the terms of an expression, it is not necessary to list the + sign at the beginning of a term.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2x + 3y - 8)</td>
<td>(-2x), (3y), -8</td>
</tr>
<tr>
<td>(3y^2 - 2x + \frac{1}{2})</td>
<td>(3y^2), (-2x), (\frac{1}{2})</td>
</tr>
<tr>
<td>(7 + x + 4 - 5x)</td>
<td>(7), (x), 4, -5x</td>
</tr>
<tr>
<td>(3(x - 1) - 4x + 2)</td>
<td>(3(x - 1)), (-4x), 2</td>
</tr>
<tr>
<td>(\frac{x + 4}{3} - 5x + 3)</td>
<td>(\frac{x + 4}{3}), (-5x), 3</td>
</tr>
</tbody>
</table>

The numerical part of a term is called its **numerical coefficient** or simply its **coefficient**. In the term 6x, the 6 is the numerical coefficient. Note that 6x means the variable \(x\) is multiplied by 6.

<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x</td>
<td>5</td>
</tr>
<tr>
<td>(-x)</td>
<td>(-1)</td>
</tr>
<tr>
<td>(\frac{2x}{3})</td>
<td>(\frac{2}{3}) since (\frac{2x}{3}) means (\frac{2}{3}x)</td>
</tr>
<tr>
<td>(\frac{x + 4}{3})</td>
<td>(\frac{1}{3}) since (\frac{x + 4}{3}) means (\frac{1}{3}(x + 4))</td>
</tr>
</tbody>
</table>

Whenever a term appears without a numerical coefficient, we assume that the numerical coefficient is 1.

4. a) What are like terms? Determine whether the following are like terms. If not, explain why.
   
   b) 3x, 4y  
   c) 7, -2  
   d) 5x², 2x  
   e) 4x, -5xy

### Exercises

**Expression:** \(-3x + 4y - 5z + 7\)

<table>
<thead>
<tr>
<th>Terms</th>
<th>Coefficient of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x</td>
<td>3</td>
</tr>
<tr>
<td>4y</td>
<td>4</td>
</tr>
<tr>
<td>-5z</td>
<td>-5</td>
</tr>
<tr>
<td>7</td>
<td>7 + Constant</td>
</tr>
</tbody>
</table>
3 Combine Like Terms

We often need to simplify expressions by combining like terms. **To combine like terms** means to add or subtract the like terms in an expression. To combine like terms, we can use the procedure that follows.

**To Combine Like Terms**

1. Determine which terms are like terms.
2. Add or subtract the coefficients of the like terms.
3. Multiply the number found in step 2 by the common variable(s).

4 Use the Distributive Property

We introduced the distributive property in Section 1.10. Because this property is so important, we will study it again. But before we do, let’s briefly review the subtraction of real numbers. Recall from Section 1.7 that

\[ 6 - 3 = 6 + (-3) \]

In general,

**Subtraction of Real Numbers**

For any real numbers \( a \) and \( b \),

\[ a - b = a + (-b) \]

We will use the fact that \( a + (-b) \) means \( a - b \) in discussing the distributive property.

**Distributive Property**

For any real numbers \( a \), \( b \), and \( c \),

\[ a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac \]

62. \[ -2(y + 8) = -2y - 16 \]

64. \[ -2(x - 4) = -2x + 8 \]

\[ 2(x + 5) = 2x + 10 \quad 2(x - 5) = 2x - 10 \]
\[ 2(x - 5) = -2x - 10 \quad -2(x - 5) = 2x + 10 \]
\[ -2(-x + 4) = 2x - 8 \quad = -2(-x) + (-2)(6) \]
\[ = (2)(-x) + (2)(4) \]
**Helpful Hint**

With a little practice, you will be able to eliminate some of the intermediate steps when you use the distributive property. When using the distributive property, there are eight possibilities with regard to signs. Study and understand the eight possibilities that follow.

<table>
<thead>
<tr>
<th>Positive Coefficient</th>
<th>Negative Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a)</strong> $2(x) = 2x$</td>
<td><strong>e)</strong> $(-2)(x) = -2x$</td>
</tr>
<tr>
<td>$2(x + 3) = 2x + 6$</td>
<td>$-2(x + 3) = -2x - 6$</td>
</tr>
<tr>
<td>$2 (+3) = +6$</td>
<td>$(-2)(+3) = -6$</td>
</tr>
<tr>
<td><strong>b)</strong> $2(x) = 2x$</td>
<td><strong>f)</strong> $(-2)(x) = -2x$</td>
</tr>
<tr>
<td>$2(x - 3) = 2x - 6$</td>
<td>$-2(x - 3) = -2x + 6$</td>
</tr>
<tr>
<td>$2(-3) = -6$</td>
<td>$(-2)(-3) = +6$</td>
</tr>
<tr>
<td><strong>c)</strong> $2(-x) = -2x$</td>
<td><strong>g)</strong> $(-2)(-x) = 2x$</td>
</tr>
<tr>
<td>$2(-x + 3) = -2x + 6$</td>
<td>$-2(-x + 3) = 2x - 6$</td>
</tr>
<tr>
<td>$2(+3) = +6$</td>
<td>$(-2)(+3) = -6$</td>
</tr>
<tr>
<td><strong>d)</strong> $2(-x) = -2x$</td>
<td><strong>h)</strong> $(-2)(-x) = 2x$</td>
</tr>
<tr>
<td>$2(-x - 3) = -2x - 6$</td>
<td>$-2(-x - 3) = 2x + 6$</td>
</tr>
<tr>
<td>$2(-3) = -6$</td>
<td>$(-2)(-3) = +6$</td>
</tr>
</tbody>
</table>

The distributive property can be expanded as follows:

$$a(b + c + d + \cdots + n) = ab + ac + ad + \cdots + an$$

**Examples of the Expanded Distributive Property**

$$3(x + y + z) = 3x + 3y + 3z$$

$$2(x + y - 3) = 2x + 2y - 6$$

80. $-3(2a + 3b - 7)$

- $-6a - 9b + 21$

84. $(8b - 1)?$

\[ 56b - 7 = \underbrace{7(8b - 1)}_7 \]

\[ 7(8b) + 7(1) \]

\[ (2 + 3 + 4) = 2(1) + 3(4) + 4(1) \]

\[ (9) = 16 + 28 + 28 \]

\[ 63 = 63 \]
5 Remove Parentheses When They Are Preceded by a Plus or Minus Sign

In the expression \((4x + 3)\), how do we remove parentheses? Recall that the coefficient of a term is assumed to be 1 if none is shown. Therefore, we may write

\[
(4x + 3) = 1(4x + 3) \\
= 1(4x) + (1)(3) \\
= 4x + 3
\]

Note that \((4x + 3) = 4x + 3\). When no sign or a plus sign precedes parentheses, the parentheses may be removed without having to change the expression inside the parentheses.

**Examples**

\[
(x + 3) = x + 3 \\
(2x - 3) = 2x - 3 \\
+(2x - 5) = 2x - 5 \\
+(x + 2y - 6) = x + 2y - 6
\]

Now consider the expression \(-(4x + 3)\). How do we remove parentheses in this expression? Here, the coefficient in front of the parentheses is \(-1\), so each term within the parentheses is multiplied by \(-1\).

\[
-(4x + 3) = -1(4x + 3) \\
= -1(4x) + (-1)(3) \\
= -4x + (-3) \\
= -4x - 3
\]

Thus, \(-(4x + 3) = -4x - 3\). When a minus sign precedes parentheses, the signs of all the terms within the parentheses are changed when the parentheses are removed.

**Examples**

\[
-(x + 4) = -x - 4 \\
-(2x + 3) = 2x - 3 \\
-(5x - y + 3) = -5x + y - 3 \\
-(-4c - 3d - 5) = 4c + 3d + 5
\]
### Simplify an Expression

Combining what we learned in the preceding discussions, we have the following procedure for **simplifying an expression**.

<table>
<thead>
<tr>
<th>To Simplify an Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Use the distributive property to remove any parentheses</td>
</tr>
<tr>
<td>2. Combine like terms</td>
</tr>
</tbody>
</table>

90. \( 7 - (2x - 9) \)

\[
\begin{align*}
7 + 6(2x - 9) \\
7 - 2x + 9 \\
-2x + 16
\end{align*}
\]

102. \(-\left(\frac{7}{8}x - \frac{1}{2}\right) - 3x\)

\[
\begin{align*}
-\frac{7}{8}x + \frac{1}{2} - 3x \\
-3\frac{1}{8}x + \frac{1}{2} \\
-3\frac{1}{8}x + \frac{1}{2}
\end{align*}
\]

108. \(4(3b - 2) - 5(c - 4) - 6b\)

\[
\begin{align*}
12b - 8 - 5c + 20 - 6b \\
6b - 5c + 12
\end{align*}
\]

114. \(2y - 6(y - 2) + 3\)

\[
\begin{align*}
2y - 6y + 12 + 3 \\
-4y + 15
\end{align*}
\]
\[
\frac{1}{5} \cdot 6 = \frac{6}{5}
\]

\[
\frac{1}{5} \cdot (x + 6) + \frac{1}{4} \cdot (4x + 12)
\]

\[
\frac{1}{5}x + 1\frac{1}{5} + 1x + 3
\]

\[
1\frac{1}{5}x + 4\frac{1}{5}
\]

\[
\frac{6}{5}x + \frac{21}{5}
\]

\[
\frac{1}{4} \cdot \frac{4}{5} > 1
\]

\[
\frac{4}{5} \cdot \frac{4}{5} = 3
\]
\[-2x + 6y - (8 + x) + (x + 8)\]