1.4 The Real Number System

We will be talking about and using various types of numbers throughout the text. This section introduces you to some of those numbers and to the structure of the real number system.

1. Identify Sets of Numbers

A set is a collection of elements listed within braces. The set \{a, b, c, d, e\} consists of five elements, namely a, b, c, d, and e. A set that contains no elements is called an empty set (or null set). The symbols \{\} or \emptyset are used to represent the empty set.

There are many different sets of numbers. Two important sets are the natural numbers and the whole numbers. The whole numbers were introduced earlier.

Nontatural numbers: \[1, 2, 3, 4, 5, \ldots\]
Whole numbers: \[0, 1, 2, 3, 4, 5, \ldots\]

Another important set of numbers is the integers.

\[\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\]

Rational numbers: [quotient of two integers, denominator not 0]

All integers are rational numbers since they can be written with a denominator of 1. For example, \(3 = \frac{3}{1}\), \(-12 = \frac{-12}{1}\), and \(0 = \frac{0}{1}\). All fractions containing integers in the numerator and denominator (with the denominator not 0) are rational numbers. For example, the fraction \(\frac{3}{5}\) is a quotient of two integers and is a rational number.

When a fraction that is a ratio of two integers is converted to a decimal number by dividing the numerator by the denominator, the quotient will always be either a terminating decimal number, such as 0.3 and 3.25, or a repeating decimal number such as 0.3333... and 5.2727... The three dots at the end of a number like 0.3333... indicate that the numbers continue to repeat in the same manner indefinitely. All terminating decimal numbers and all repeating decimal numbers are rational numbers which can be expressed as a quotient of two integers. For example, \(0.3 = \frac{3}{10}\), \(3.25 = \frac{325}{100}\), \(0.3333... = \frac{1}{3}\)

Most of the numbers that we use are rational numbers. However, some numbers are not rational. Numbers such as the square root of 2, written \(\sqrt{2}\), are not rational numbers. Any number that can be represented on the number line that is not a rational number is called an irrational number. Irrational numbers are non-terminating, non-repeating decimal numbers. For example, \(\sqrt{2}\) cannot be expressed exactly as a decimal number. Irrational numbers can only be approximated by decimal numbers. \(\sqrt{2}\) is approximately 1.41. Thus, we may write \(\sqrt{2} \approx 1.41\). Some irrational numbers are illustrated on the number line in Figure 1.12. Rational and irrational numbers will be discussed further in later chapters.

\[\text{FIGURE 1.12}\]

Notice that many different types of numbers can be illustrated on a number line. Any number that can be represented on the number line is a real number.

Real numbers: [all numbers that can be represented on a real number line]
Real #s

- \( \ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \)

(Counting #s)

Natural Numbers \( \mathbb{N} \) \( \{1, 2, 3, \ldots\} \)

Whole #s \( \mathbb{W} \) \( \{0, 1, 2, 3, \ldots\} \)

Integers \( \mathbb{Z} \) \( \{ \ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\} \)

Rational #s

Fractions

\[
\frac{2}{3}, \frac{7}{8}, \frac{3}{4}, \frac{6}{1} = 6
\]

Terminating

\[\frac{1}{3} = .2\]

\[\frac{1}{8} = .125\]

Repeating Decimals

\[.25 = \frac{25}{100} = \frac{1}{4}\]

\[.\overline{3} = \frac{1}{3}\]

Irrational

Non-repeating - Non-terminating

Decimal

\[2.14144144414444\ldots\]

\[\sqrt{2}, \sqrt{7}, 3\sqrt{9}\]

\[\pi \approx 3.14159\]

Real # \( \rightarrow \) Number Line

Includes Rational \& Irrational
Jenna Webber's test grades are 78, 93, 57, 72, and 72. For Jenna's grades, determine the (a) mean and (b) median.

\[
\text{Mean} = \text{Average} = \frac{78 + 93 + 57 + 72 + 72}{5} = \frac{350}{5} = 70
\]

\[
\text{Median} \quad \text{Middle \#} \quad 57 \quad 72 \quad 72 \quad 78 \quad 93
\]

For the odd number of scores, the median is the middle number:

\[
\text{Median} = 72
\]

For the even number of scores, the median is the average of the two middle numbers:

\[
\text{Median} = \frac{76 + 78}{2} = 77
\]
FIGURE 1.13

Real numbers

Rational numbers

Integers

Noninteger rational numbers

Negative integers

Whole numbers

0

Natural numbers

(b)

Real Numbers

Rational numbers

(Integers and noninteger rational numbers)

-12 4 0

\( \frac{3}{8} \) \( \frac{1}{3} \) -1.24

-1\( \frac{3}{5} \) -2.463

Irrational numbers

(Certain square roots and other special numbers)

\( \sqrt{2} \) \( \sqrt{5} \)

\( \pi \) \( \sqrt{12} \)

2.4149414146...

*Other higher roots like \( \sqrt[3]{2} \) and \( \sqrt[4]{5} \) are also irrational numbers.
52. Consider the following set of numbers.
\[ \left\{ -6, 7, 12.4, -\frac{9}{5}, -2 \frac{1}{4}, \sqrt{3}, 0, 9, \sqrt[3]{7}, 0.35 \right\} \]
\[ \overline{0.3}, 0.1\overline{3}, 3 \overline{3} \ldots \]

\[ \text{Natural #s} \ \{ 7, 9, 3 \} \]
\[ \text{Whole #s} \ \{ 0, 7, 9, 3 \} \]
\[ \text{Integers} \ \{ -6, 7, 0, 9, 3 \} \]
\[ \text{Rational #s} \ \{ -6, 7, 12.4, -\frac{9}{3}, -2.4, 0.9, 35, \overline{3} \} \]
\[ \text{Irrational #s} \ \{ \sqrt{3}, \sqrt{7}, 2.1\overline{3}, 3 \overline{3} \ldots \} \]
\[ \text{Real #s} \ \text{ALL} \]
Consider the following set of numbers.

\[
\left\{-9, -\frac{3}{5}, 0, 0.1, \sqrt{2}, 7.6, \sqrt{36}\right\}
\]

\[\overline{0.274}, 3.15115\ldots\]