1.6 Addition of Real Numbers

There are many practical uses for negative numbers. A submarine diving below sea level, a bank account that has been overdrawn, a business spending more than it earns, and a temperature below zero are some examples.

The four basic operations of arithmetic are addition, subtraction, multiplication, and division. In the next few sections, we will explain how to add, subtract, multiply, and divide numbers. We will consider both positive and negative numbers. In this section, we discuss the operation of addition.

1 Add Real Numbers Using a Number Line

\[
\begin{align*}
4 + 7 &= 11 \\
-4 + 7 &= 3 \\
4 + (-7) &= -3 \\
(-4) + (-7) &= -11
\end{align*}
\]

Take the difference of the 2 numbers, give the answer the sign of the number with the larger absolute value.
\[
\begin{align*}
6 + (-2) &= 4 \\
-6 + 2 &= -4 \\
-6 + (-2) &= -8 \\
6 + 2 &= 8 \\
8 + (-3) &= 5 \\
-8 + (-3) &= -11 \\
8 + 3 &= 11 \\
-8 + 3 &= -5 \\
\frac{3}{10} + \frac{4}{10} &= \frac{7}{10} \\
\frac{3}{10} + (-\frac{4}{10}) &= -\frac{1}{10} \\
\frac{3}{10} + (-\frac{4}{10}) &= -\frac{1}{10} \\
-\frac{3}{10} + \frac{4}{10} &= \frac{1}{10} \\
9 + 11 &= 20 \\
9 + (-11) &= -2 \\
-9 + 11 &= 2 \\
-9 + (-11) &= -20
\end{align*}
\]
3 Identify Opposites

Now let’s consider **opposites**, or **additive inverses**.

**Opposites (or Additive Inverses)**

Any two numbers whose sum is zero are said to be **opposites** (or **additive inverses**) of each other. In general, if we let \( a \) represent any real number, then its opposite is \(-a\) and \( a + (-a) = 0\).

In Example 4, the sum of 5 and \(-5\) is 0. Thus, \(-5\) is the opposite of 5 and 5 is the opposite of \(-5\).

**EXAMPLE 8** Find the opposite of each number. a) 3 b) \(-4\) c) \(-\frac{7}{8}\)

\[
\begin{align*}
-3 + \frac{3}{8} &= 0 \\
8 + \frac{-8}{3} &= 0 \\
\frac{-7}{8} + \frac{7}{8} &= 0 \\
-9 + \frac{9}{3} &= 0 \\
\frac{6}{7} + \frac{-6}{7} &= 0 \\
\frac{-3}{5} + \frac{3}{5} &= 0
\end{align*}
\]
4 Add Using Absolute Values

Now that we have had some practice adding signed numbers on a number line, we give a rule (in two parts) for using absolute value to add signed numbers. Remember that the absolute value of a nonzero number will always be positive. The first part of the rule follows.

Adding Real Numbers with the Same Sign

To add real numbers with the same sign (either both positive or both negative), add their absolute values. The sum has the same sign as the numbers being added.

**EXAMPLE 9** Add 4 + 8.

*Solution* Since both numbers have the same sign, both positive, we add their absolute values: \(|4| + |8| = 4 + 8 = 12\). Since both numbers being added are positive, the sum is positive. Thus, \(4 + 8 = 12\).

Now Try Exercise 49

**EXAMPLE 10** Add \(-6 + (-9)\).

*Solution* Since both numbers have the same sign, both negative, we add their absolute values: \(|-6| + |-9| = 6 + 9 = 15\). Since both numbers being added are negative, their sum is negative. Thus, \(-6 + (-9) = -15\).

Now Try Exercise 51

The sum of two positive numbers will always be positive and the sum of two negative numbers will always be negative.

Adding Two Signed Numbers with Different Signs

To add two signed numbers with different signs (one positive and the other negative), subtract the smaller absolute value from the larger absolute value. The answer has the sign of the number with the larger absolute value.
16. 3
20. $\frac{-1}{4}$
24. $-0.721$

28. $9 + (-12) = -3$
32. $-8 + 8 = 0$
36. $6 + (-5) = 1$
40. $0 + (-3) = -3$
44. $-9 + 13 = 4$
48. $-27 + (-9) = -36$
52. $-25 + (-36) = -61$
56. $34 + (-40) = -6$
60. $-33 + (-92)$

76. $\frac{2}{9} + \frac{3}{10}$
80. $\frac{8}{9} + \left( -\frac{1}{3} \right)$
84. $-\frac{1}{15} + \left( -\frac{5}{6} \right)$

\[
\begin{align*}
\frac{8}{9} &= \frac{8}{9} \\
+ \frac{-1}{3} \times \frac{3}{3} &= \frac{-3}{9} \\
\left( \frac{\text{黄}}{9} \right)
\end{align*}
\]

\[
\frac{\text{红}}{30} \]

\[
\begin{align*}
-\frac{1}{15} \cdot \frac{2}{2} &= \frac{-2}{30} \\
+ \frac{-5}{6} \cdot \frac{5}{5} &= \frac{-25}{30} \\
\frac{-27}{30}
\end{align*}
\]