$$-\left(\frac{3}{4}x - \frac{1}{3}\right) + 2x$$

$$= -\frac{3}{4}x + \frac{1}{3} + 2x$$

$$= \frac{5}{4}x + \frac{1}{3}$$
\[
\frac{4x}{3} = \frac{12}{4} \quad \frac{-5x}{-5} = \frac{30}{-5}
\]
\[x = 3 \quad x = -6\]

\[
\frac{3}{2} \left[ \frac{\frac{2}{3}x}{1} = \frac{16}{1} \right] \frac{3}{2}
\]
\[x = 24\]
\[
\frac{\frac{8}{3}x}{1} = 16
\]

\[
\frac{1}{5} \left[ \frac{5x}{1} = \frac{-2}{15} \right] \frac{1}{5}
\]
\[x = \frac{-2}{75}\]

\[
\frac{-x}{-1} = \frac{4}{1}
\]
\[x = -4\]

\[
\frac{-x}{-1} = \frac{7}{1}
\]
\[x = 7\]

\[
\frac{-x}{-5} = \frac{-4}{1}
\]
\[x = 20\]
2.4 Solving Linear Equations with a Variable on Only One Side of the Equation

1. Solve linear equations with a variable on only one side of the equal sign.
2. Solve equations containing decimal numbers or fractions.

1 Solve Linear Equations with a Variable on Only One Side of the Equal Sign

In this section, we discuss how to solve linear equations using both the addition and multiplication properties of equality when a variable appears on only one side of the equal sign. In Section 2.5, we will discuss how to solve linear equations using both properties when a variable appears on both sides of the equal sign.

The general procedure we use to solve equations is to "isolate the variable." That is, get the variable alone on one side of the equal sign.

1 Solve Equations with the Variable on Both Sides of the Equal Sign

The equation $4x + 6 = 2x + 4$ contains the variable $x$ on both sides of the equal sign. To solve equations of this type, we must use the appropriate properties to rewrite the equation with all terms containing the variable on only one side of the equal sign and all terms not containing the variable on the other side of the equal sign. Following is a general procedure, similar to the one outlined in Section 2.4, that can be used to solve linear equations with the variable on both sides of the equal sign. The steps in the procedure are only guidelines to use. For example, there may be times when you may choose to use the distributive property, step 2, before multiplying both sides of the equation by the LCD, step 1. We will illustrate this in Examples 8 and 9.

| To Solve Linear Equations with the Variable on Both Sides of the Equal Sign |
|---|---|
| 1. If the equation contains fractions, multiply both sides of the equation by the least common denominator. This will eliminate fractions from the equation. |
| 2. Use the distributive property to remove parentheses. |
| 3. Combine like terms on the same side of the equal sign. |
| 4. Use the addition property to rewrite the equation with all terms containing the variable on one side of the equal sign and all terms not containing the variable on the other side of the equal sign. It may be necessary to use the addition property twice to accomplish this goal. You will eventually get an equation of the form $ax = b$. |
| 5. Use the multiplication property to isolate the variable. This will give a solution of the form $x = \text{some number}$. |
| 6. Check the solution in the original equation. |
16. \(2x - 4 = 8\)

\[
\begin{align*}
+4 & +4 \\
\hline
2x & = 12 \\
\frac{2x}{2} & = \frac{12}{2} \\
x & = 6 \\
2(6) & - 4 = 8\ 
\end{align*}
\]

24. \(-4x - 7 = -6\)

\[
\begin{align*}
+7 & +7 \\
\hline
-4x & = 1 \\
\frac{-4x}{-4} & = \frac{1}{-4} \\
x & = -\frac{1}{4} \\
-\frac{1}{4} \cdot \frac{1}{4} \cdot -7 & = -6 \\
1 \cdot -7 & = -6
\end{align*}
\]

38. \(15 = 6x - 3 + 3x\)

\[
\begin{align*}
15 & = 9x - 3 \\
+3 & +3 \\
\hline
18 & = 9x \\
\frac{18}{9} & = \frac{9x}{9} \\
x & = 2
\end{align*}
\]

66. \(5(3x + 1) - 12x = -2\)

\[
\begin{align*}
5[\frac{3 \cdot \frac{7}{3} + 1}{1}] & - 12 \cdot \frac{7}{3} \cdot 2 \\
15x + 5 - 12x & = -2 \\
3x + 5 & = -2 \\
-5 & -5 \\
\hline
3x & = -\frac{7}{3} \\
x & = -\frac{7}{3} = -2\frac{1}{3} \\
5(6) & + 28 = 2 \ \\
-7 + 28 & = 2
\end{align*}
\]

70. \(3y - (y + 5) = 9\)

\[
\begin{align*}
3y & - y - 5 = 9 \\
2y & - 5 = 9 \\
+5 & +5 \\
\hline
2y & = 14 \\
\frac{2y}{2} & = \frac{14}{2} \\
y & = 7
\end{align*}
\]

100. \(\frac{1}{3}x - \frac{3}{4}x = \frac{1}{5}\)

\[
\begin{align*}
60 & \left[\frac{1}{3}x - \frac{3 \cdot 15}{4}x = \frac{120}{5}\right] \\
20x - 45x & = 12 \\
-25x & = \frac{12}{-25} \\
\frac{-25x}{-25} & = \frac{12}{-25} \\
x & = -\frac{12}{25} \\
\text{Check:} \\
\frac{1}{3}x - \frac{3}{4}x & = \frac{1}{5} \\
\text{LCD = 60}
\end{align*}
\]

\[\boxed{4}\]
42. \(0.15 = 0.05x - 1.35 - 0.20x\)

\[
\begin{align*}
100 \left(0.15 &= 0.05x - 1.35 - 0.20x\right) \\
15 &= 5x - 135 - 20x \\
15 &= -15x - 135 \\
15 &= -15x - 135 \\
150 &= -15x \\
-10 &= x
\end{align*}
\]

46. \(\frac{m - 6}{5} = 2\)

\[
\begin{align*}
\frac{m - 6}{5} &= 2 \\
\frac{(m - 6)}{5} &= \frac{2}{1} \\
5 \left(\frac{m - 6}{5}\right) &= \frac{2}{1} \\
m - 6 &= 10 \\
m &= 16
\end{align*}
\]

90. \(\frac{x}{4} - 6x = 23\)

\[
\begin{align*}
\frac{1}{4} \left(\frac{x}{4} - 6x\right) &= 23 \\
x - 24x &= 92 \\
-23x &= \frac{92}{-23} \\
x &= -4
\end{align*}
\]
**Helpful Hint**

Some of the most commonly used terms in algebra are “evaluate,” “simplify,” “solve,” and “check.” Make sure you understand what each term means and when each term is used.

**Evaluate:** To evaluate an expression means to find its numerical value.

Evaluate

\[ 16 \div 2^2 + 36 \div 4 \]
\[ = 16 \div 4 + 36 \div 4 \]
\[ = 4 + 36 \div 4 \]
\[ = 4 + 9 \]
\[ = 13 \]

Evaluate

\[-x^2 + 3x - 2 \text{ when } x = 4\]
\[ = -4^2 + 3(4) - 2 \]
\[ = -16 + 12 - 2 \]
\[ = -4 - 2 \]
\[ = -6 \]

**Simplify:** To simplify an expression means to perform the operations and combine like terms.

Simplify

\[ 3(x - 2) - 4(2x + 3) \]
\[ = 3x - 6 - 8x - 12 \]
\[ = -5x - 18 \]

Note that when you simplify an expression containing variables you do not generally end up with just a numerical value unless all the variable terms happen to add to zero.

**Solve:** To solve an equation means to find the value or the values of the variable that make the equation a true statement.

Solve

\[ 2x + 3(x + 1) = 18 \]
\[ 2x + 3x + 3 = 18 \]
\[ 5x + 3 = 18 \]
\[ 5x = 15 \]
\[ x = 3 \]

**Check:** To check the proposed solution to an equation, substitute the value in the original equation. If this substitution results in a true statement, then the answer checks. For example, to check the solution to the equation just solved, we substitute 3 for \( x \) in the original equation.

Check

\[ 2x + 3(x + 1) = 18 \]
\[ 2(3) + 3(3 + 1) \]
\[ 2(3) + 3(4) \]
\[ 6 + 12 \]
\[ 18 = 18 \] True

Since we obtained a true statement, the 3 checks.

It is important to realize that expressions may be evaluated or simplified (depending on the type of problem) and equations are solved and then checked.