4.2 Negative Exponents

1 Understand the Negative Exponent Rule

One additional rule that involves exponents is the negative exponent rule. You will need to understand negative exponents to be successful with scientific notation in the next section.

The negative exponent rule will be developed using the quotient rule illustrated in Example 1.

**EXAMPLE 1** Simplify \(\frac{x^3}{x^2}\) by a) using the quotient rule and b) dividing out common factors.

**Solution**

a) By the quotient rule,
\[
\frac{x^3}{x^2} = x^{3-2} = x
\]

b) By dividing out common factors,
\[
\frac{x^3}{x^2} = \frac{x \cdot x \cdot x}{x \cdot x} = \frac{1}{x^2}
\]

In Example 1, we see that \(\frac{x^3}{x^2}\) is equal to both \(x\) and \(\frac{1}{x^2}\). Therefore, \(x\) must equal \(\frac{1}{x^2}\). That is, \(x^{-2} = \frac{1}{x^2}\). This is an example of the **negative exponent rule**.

**Negative Exponent Rule**

\[x^{-m} = \frac{1}{x^m}, \quad x \neq 0\]

When a variable or number is raised to a negative exponent, the expression may be rewritten as 1 divided by the variable or number raised to that positive exponent.

**Examples**

\[
x^{-6} = \frac{1}{x^6} \quad \quad 4^{-2} = \frac{1}{4^2} = \frac{1}{16}
\]

\[
y^{-7} = \frac{1}{y^7} \quad \quad 5^{-3} = \frac{1}{5^3} = \frac{1}{125}
\]

38. \(\frac{x^{-2}}{x^5}\)

24. \((a^7)^{-4} = \frac{1}{a^{7 \cdot 4}} = \frac{1}{a^{28}}\)

26. \((x^{-9})^{-2}\)

\[
\frac{x^{-2}}{x^5} = x^{-2-5} = x^{-7} = \frac{1}{x^7}
\]

\[
\left(\frac{x^{-2}}{x^5}\right)^2 = \frac{1}{x^{5+2}} = \frac{1}{x^7}
\]
EXAMPLE 2 ➢ Use the negative exponent rule to write each expression with a positive exponent. Simplify the expressions further when possible.

a) \(y^{-5}\)  
b) \(x^{-4}\)  
c) \(2^{-3}\)  
d) \(6^{-1}\)  
e) \(-5^{-3}\)  
f) \((-5)^{-3}\)

Solution

\[-5^{-3} = \frac{-1}{5^3}\]

\[= \frac{-1}{125}\]

\[(-5)^{-3} = \frac{1}{(-5)^3}\]

\[= \frac{1}{-125}\]

EXAMPLE 3 ➢ Use the negative exponent rule to write each expression with a positive exponent.

a) \(\frac{1}{x^2}\)  
b) \(\frac{1}{4^{-1}}\)

26. \((x^{-2})^{-2}\)  

52. \(x^{-3} \cdot x^{-5}\)

86. \(\frac{x^6}{x^7}\)

90. \(2x^{-1}y\)

94. \(-5x^4y^{-3}\)
This example illustrates that \( \left( \frac{a}{b} \right)^{-m} = \left( \frac{b}{a} \right)^m \) when \( a \neq 0 \) and \( b \neq 0 \). Thus, for example, 
\[
\left( \frac{3}{4} \right)^{-5} = \left( \frac{4}{3} \right)^5 \quad \text{and} \quad \left( \frac{5}{9} \right)^{-3} = \left( \frac{9}{5} \right)^3.
\]
We can summarize this information as follows.

**A Fraction Raised to a Negative Exponent Rule**

For a fraction of the form \( \frac{a}{b} \), \( a \neq 0 \) and \( b \neq 0 \), 
\[
\left( \frac{a}{b} \right)^{-m} = \left( \frac{b}{a} \right)^m.
\]

**EXAMPLE 10**  Simplify.  
\[\text{a) } \left( \frac{4}{5} \right)^{-3} \quad \text{b) } \left( \frac{x^5}{y^3} \right)^4\]
### Summary of Rules of Exponents

1. $x^m \cdot x^n = x^{m+n}$ \hspace{1cm} \text{product rule}
2. $\frac{x^m}{x^n} = x^{m-n}, \quad x \neq 0$ \hspace{1cm} \text{quotient rule}
3. $x^0 = 1, \quad x \neq 0$ \hspace{1cm} \text{zero exponent rule}
4. $(x^m)^n = x^{mn}$ \hspace{1cm} \text{power rule}
5. $\left( \frac{a}{b} \right)^m = \frac{a^m}{b^m}, \quad b \neq 0, y \neq 0$ \hspace{1cm} \text{expanded power rule}
6. $x^{-m} = \frac{1}{x^m}, \quad x \neq 0$ \hspace{1cm} \text{negative exponent rule}
7. $\left( \frac{a}{b} \right)^{-m} = \left( \frac{b}{a} \right)^m, \quad a \neq 0, b \neq 0$ \hspace{1cm} \text{a fraction raised to a negative exponent rule}