### 4.3 Scientific Notation

1. Convert numbers to and from scientific notation.
2. Recognize numbers in scientific notation with a coefficient of 1.
3. Do calculations using scientific notation.

#### Convert Numbers to and from Scientific Notation

We often see, and sometimes use, very large or very small numbers. For example, in September 2006, the world population was about 6,539,000,000 people. You may have heard that an influenza virus is about 0.0000001 meters in diameter. Because it is difficult to work with many zeros, we can express such numbers using exponents. For example, the number 6,539,000,000 could be written $6.539 \times 10^8$ and the number $0.0000001$ could be written $1.0 \times 10^{-7}$.

Numbers such as $6.539 \times 10^8$ and $1.0 \times 10^{-7}$ are in a form called **scientific notation**. Each number written in scientific notation is written as a number greater than or equal to 1 and less than 10 ($1 \leq a < 10$) multiplied by some power of 10. The exponent on the 10 must be an integer.

#### Examples of Numbers in Scientific Notation

$$
\begin{align*}
3.84 \times 10^4 & , & 3.762 \times 10^3 & , & 3.262 \times 10^2 & , & 8.07 \times 10^{-3} & , & 1.0 \times 10^{-5} \\
\end{align*}
$$

Below we change the number 68,400 to scientific notation.

$$
68,400 = 6.84 \times 10^4
$$

*Note that $10^4 = 10 \times 10 \times 10 = 10^3$."

#### To Write a Number in Scientific Notation

1. Move the decimal point in the original number to the right of the first nonzero digit. This will give a number greater than or equal to 1 and less than 10.
2. Count the number of places you moved the decimal point to obtain the number in step 1. If the original number was 10 or greater, the count is considered positive. If the original number was less than 1, the count is considered negative.
3. Multiply the number obtained in step 1 by 10 raised to the count (power) found in step 2.

#### Example 1

Write the following numbers in scientific notation.

- a) $18,500 = 1.85 \times 10^4$
- b) $0.0000416 = 4.16 \times 10^{-5}$
- c) $3,721,000 = 3.721 \times 10^6$
- d) $0.0093 = 9.3 \times 10^{-3}$

#### Exercises

16. $0.000089$
20. $0.0000186$
24. $74,100$
28. $416,000$

$$
\begin{align*}
0.000089 &= 8.9 \times 10^{-5} \\
0.0000186 &= 1.86 \times 10^{-6} \\
74,100 &= 7.41 \times 10^4 \\
416,000 &= 4.16 \times 10^5
\end{align*}
$$
To Convert a Number from Scientific Notation to Decimal Form

1. Observe the exponent of the power of 10.

2. a) If the exponent is positive, move the decimal point in the number (greater than or equal to 1 and less than 10) to the right the same number of places as the exponent. It may be necessary to add zeros to the number. This will result in a number greater than or equal to 10.

   b) If the exponent is 0, do not move the decimal point. Drop the factor $10^0$ since it equals 1. This will result in a number greater than or equal to 1 but less than 10.

   c) If the exponent is negative, move the decimal point in the number to the left the same number of places as the exponent (dropping the negative sign). It may be necessary to add zeros to the number. This will result in a number less than 1.

**EXAMPLE 2**

Write each number without exponents.

(a) $2.9 \times 10^4$

(b) $6.28 \times 10^{-3}$

(c) $7.95 \times 10^8$  

$2,9000$  

$0.00628$  

$7,750,000,000$  

$6.15 \times 10^5 = 615,000$

$4.6 \times 10^1$

$3.14 \times 10^{-1}$

$7.13 \times 10^{-4} = 0.000713$
2 Recognize Numbers in Scientific Notation with a Coefficient of 1

We often hear terms like kilograms, milligrams, and gigabytes. For example, an aspirin tablet bottle may indicate that each aspirin contains 325 milligrams of aspirin. Your hard drive on your computer may hold 40 gigabytes of memory. The prefixes kilo, milli, and giga are some of the prefixes used in the metric system. The metric system is used in every Westernized nation except the United States as the main system of measurement. The prefixes are always used with some type of base unit. The base unit may be meters, m (a unit of length); gram, g (a unit of mass); liter, l (a unit of volume); bits, b (a unit of computer memory); or hertz, Hz (a measure of frequency).

For example, a millimeter is \( \frac{1}{1000} \) meter. A megagram is 1,000,000 grams, and so on. The following table illustrates the meaning of some prefixes.*

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Meaning</th>
<th>Symbol</th>
<th>Meaning as a Decimal Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>nano</td>
<td>( 10^{-9} )</td>
<td>n</td>
<td>( \frac{1}{1,000,000,000} ) or 0.000000001</td>
</tr>
<tr>
<td>micro</td>
<td>( 10^{-6} )</td>
<td>( \mu )</td>
<td>( \frac{1}{1,000,000} ) or 0.000001</td>
</tr>
<tr>
<td>milli</td>
<td>( 10^{-3} )</td>
<td>m</td>
<td>( \frac{1}{1000} ) or 0.001</td>
</tr>
<tr>
<td>base unit**</td>
<td>( 10^{0} )</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>kilo</td>
<td>( 10^{3} )</td>
<td>k</td>
<td>1000</td>
</tr>
<tr>
<td>mega</td>
<td>( 10^{6} )</td>
<td>M</td>
<td>1,000,000</td>
</tr>
<tr>
<td>giga</td>
<td>( 10^{9} )</td>
<td>G</td>
<td>1,000,000,000</td>
</tr>
</tbody>
</table>

*The base unit is not a prefix. We included this row to include \( 10^0 \) in the chart.

In Figure 4.1, we see that the frequency of FM radio and VHF TV is about \( 10^8 \) hertz (or cycles per second). Thus, the frequency of FM radio is \( 10^8 = 1.0 \times 10^8 = 100,000,000 \) hertz. This number, one hundred million hertz, can also be expressed as \( 100 \times 10^6 \) or 100 megahertz, 100 MHz.
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14. \( 3.61 \times 10^6 \)
20. \( \frac{0.00000186}{10^{-6}} = \frac{1}{1,000,000} \)
34. \(7.26 \times 10^{-6} = 0.0000726 = 4.21 \times 10^{18}\)
56. \((1.3 \times 10^{-8})(1.74 \times 10^6)\)

\[
(1.3)(1.74) \times 10^{-8} \times 10^6 \\
= 2.262 \times 10^{-2} \\
= 0.02262
\]

60. \(\frac{6.0 \times 10^{-3}}{3.0 \times 10^1}\)

\[
\frac{6}{3} \times \frac{10^{-3}}{10^1} \\
= 2 \times 10^{-4}
\]

68. \((67,000)(200,000)\)

\[
(6.7 \times 10^4)(2.0 \times 10^5) = (6.7)(2.0) (10^4)(10^5)
\]

\[
= 1.34 \times 10^9
\]

70. \(\frac{0.00004}{200}\)

\[
= 0.000002
\]

True Scientific Notation