4.5 Multiplication of Polynomials

1. Multiply a Monomial by a Monomial

We begin our discussion of multiplication of polynomials by multiplying a monomial by a monomial. To multiply two monomials, multiply their coefficients and use the product rule of exponents to determine the exponents on the variables. Problems of this type were done in Section 4.1.

Example 1. Multiply.

a) \((7x^2)(6x^3)\)  
b) \((4b^3)(-9b^7)\)

Solution

a) \((7x^2)(6x^3) = 7 \cdot 6 \cdot x^2 \cdot x^3 = 42x^{2+3} = 42x^5\)
b) \((4b^3)(-9b^7) = (4)(-9) \cdot b^3 \cdot b^7 = -36b^{3+7} = -36b^{10}\)

Now Try Exercise 15

16. \((-7p^5)(-2p^3) = 14p^8\)

20. \((4a^3b^7)(6a^2b)\)

24. \(\frac{3}{4}x(8x^2y^3)\)

\[\left(\frac{3}{4}\right)(8)(x^1)(x^2)(y^3)\]

\(= 6x^3y^3\)

\[\frac{3}{4} \cdot \frac{8}{6} = \frac{24}{4} = \frac{6}{6}\]
Multiply a Polynomial by a Monomial

To multiply a polynomial by a monomial, we use the distributive property presented earlier.

\[ a(b + c) = ab + ac \]

30. \(-4p(-3p + 6)\)

\[ 12p^2 - 24p \]

\[(3x^2 + x - 6)x\]

\[ 3x^3 + x^2 - 6x \]

34. \(-6c(-3c^2 + 5c - 6)\)

36. \((3x^2 + x - 6)x\)

\[ -6c(-3c^2 + 5c - 6) \]

\[ 18c^3 - 30c^2 + 36c \]
### Multiply Binomials Using the Distributive Property

Now we will discuss multiplying a binomial by a binomial. Before we explain how to do this, consider the multiplication problem $43 \cdot 12$.

<table>
<thead>
<tr>
<th>43</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>516</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Note how the 2 multiplies both the 3 and the 4, and the 1 also multiplies both the 3 and the 4. That is, every digit in the multiplier multiplies every digit in the multiplicand. We can also illustrate the multiplication process as follows:

\[
(43)(12) = (40 + 3)(10 + 2)
\]

\[
= (40)(10) + (40)(2) + (3)(10) + (3)(2)
\]

\[
= 400 + 80 + 30 + 6
\]

\[
= 516
\]

Whenever any two polynomials are multiplied, the same process must be followed. That is, every term in one polynomial must multiply every term in the other polynomial.

Consider multiplying $(a + b)(c + d)$. Treating $(a + b)$ as a single term and using the distributive property, we get

\[
(a + b)(c + d) = (a + b)c + (a + b)d
\]

Using the distributive property a second time gives

\[
= ac + bc + ad + bd
\]

Notice how each term of the first polynomial was multiplied by each term of the second polynomial, and all the products were added to obtain the answer.

**EXAMPLE 9** Multiply $(3x + 2)(x - 5)$.

**Solution**

\[
(3x + 2)(x - 5) = (3x)(x) + (3x)(-5) + (2)(x) + (2)(-5)
\]

\[
= 3x^2 - 15x + 2x - 10
\]

\[
= 3x^2 - 13x - 10
\]

> **Now Try Exercise 43**

**EXAMPLE 10** Multiply $(x - 4)(y + 3)$.

**Solution**

\[
(x - 4)(y + 3) = (x - 4)y + (x - 4)3
\]

\[
= xy - 4y + 3x - 12
\]

\[
\text{\[3\times57=\fbox{191}\]}
\]

\[
\text{\[\fbox{40+5}\times50+2\]} = \]

\[
\text{\[\fbox{2000}+2200+150+21\]} = \]

\[
\text{\[\fbox{2451}\]}
\]

\[
\text{\[2\frac{3}{4}\times1\frac{1}{2}=\fbox{\frac{27}{8}}\]}
\]

\[
\text{\[\frac{2}{3}\times\frac{3}{4} = \frac{1}{2}\]}
\]

\[
\text{\[\frac{4\frac{3}{4}}{\frac{3}{4}+\frac{3}{8}+\frac{3}{2}} = 4\frac{8}{3}\]}
\]

\[
\text{\[2+(\frac{3}{8})\]}
\]

\[
\text{\[\frac{2}{3}+\frac{3}{4}+\frac{3}{8}\]}
\]

\[
\text{\[3\left(\frac{3}{8}+\frac{3}{2}\right) = 4\frac{8}{3}\]}
\]
# Multiply Binomials Using the FOIL Method

A common method used to multiply two binomials is the **FOIL method**. This procedure also results in each term of one binomial being multiplied by each term in the other binomial. Students often prefer to use this method when multiplying two binomials.

The FOIL method is not actually a different method used to multiply binomials but rather an acronym to help students remember to correctly apply the distributive property. We could have used **IFOL** or any other arrangement of the four letters. However, FOIL is easier to remember than the other arrangements.

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### The FOIL Method

Consider \((a + b)(c + d)\).

- **F** stands for first—multiply the first terms of each binomial together:

  \[(a + b)(c + d)\quad \text{product } ac\]

- **O** stands for outer—multiply the two outer terms together:

  \[(a + b)(c + d)\quad \text{product } ad\]

- **I** stands for inner—multiply the two inner terms together:

  \[(a + b)(c + d)\quad \text{product } bc\]

- **L** stands for last—multiply the last terms together:

  \[(a + b)(c + d)\quad \text{product } bd\]

The product of the two binomials is the sum of these four products.

\[(a + b)(c + d) = ac + ad + bc + bd\]
Multiply Binomials Using Formulas for Special Products

Example 13 illustrates a special product, the product of the sum and difference of the same two terms.

**Product of the Sum and Difference of the Same Two Terms**

\[(a + b)(a - b) = a^2 - b^2\]

In this special product, \(a\) represents one term and \(b\) the other term. Then \((a + b)\) is the sum of the terms and \((a - b)\) is the difference of the terms. This special product is also called the **difference of two squares formula** because the expression on the right side of the equal sign is the difference of two squares. Since multiplication is commutative \((a + b)(a - b) = a^2 - b^2\) can also be written \((a - b)(a + b) = a^2 - b^2\).

**EXAMPLE 14** Use the rule for finding the product of the sum and difference of two quantities to multiply each expression.

a) \((x + 5)(x - 5)\)  

\[
(x+5)(x-5) = x^2 - 5x + 5x - 25 = x^2 - 25
\]

b) \((2x + 4)(2x - 4)\)

\[
(2x+4)(2x-4) = 4x^2 - 8x + 8x - 16 = 4x^2 - 16
\]

c) \((3x - 2y)(3x + 2y)\)

\[
(3x-2y)(3x+2y) = 9x^2 + 6xy - 6xy - 4y^2 = 9x^2 - 4y^2
\]

80. \((2x - 7)(2x + 7)\)

88. \((4 + 3w)(4 - 3w)\)
Example 15 illustrates the **square of a binomial**, another special product.

**Square of Binomial Formulas**

\[
(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2
\]

\[
(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2
\]

To square a binomial, add the square of the first term, twice the product of the terms, and the square of the second term.

92. \((7a + 2)^2\)

\[
(7a + 2)^2 = (7a + 2)(7a + 2) = 49a^2 + 28a + 4
\]

\[
(6a - 5)^2 = (6a - 5)(6a - 5) = 36a^2 - 60a + 25
\]
6 Multiply Any Two Polynomials

When multiplying a binomial by a binomial, we saw that every term in the first binomial was multiplied by every term in the second binomial. When multiplying any two polynomials, each term of one polynomial must be multiplied by each term of the other polynomial. In the multiplication \((3x + 2)(4x^2 - 5x - 3)\), we use the distributive property as follows:

\[
(3x + 2)(4x^2 - 5x - 3) \\
= 3x(4x^2 - 5x - 3) + 2(4x^2 - 5x - 3) \\
= 12x^3 - 15x^2 - 9x + 8x^2 - 10x - 6 \\
= 12x^3 - 7x^2 - 19x - 6
\]

Thus, \((3x + 2)(4x^2 - 5x - 3) = 12x^3 - 7x^2 - 19x - 6\).

96. \((x - 1)(3x^2 + 3x + 2)\) 6 terms

\[
3x^3 + 3x^2 + 2x \\
-3x^2 - 3x - 2 \\
3x^2 - x - 2
\]

104. \((6x + 4)(2x^2 + 2x - 4)\)
118. Consider the figure below.

\[ \begin{array}{cc}
  a & b \\
  a & \\
  b & 
\end{array} \]

a) Write an expression for the length of the top.
b) Write an expression for the length of the left side.
c) Is this figure a square? Explain.
d) Express the area of this square as the square of a binomial.