5.7 Applications of Quadratic Equations

1. Solve applications by factoring quadratic equations.
2. Learn the Pythagorean Theorem.

1. Solve Applications by Factoring Quadratic Equations

In Section 5.6, we learned how to solve quadratic equations by factoring. In this section, we will discuss and solve application problems that require solving quadratic equations to obtain the answer. In Example 1, we will solve a problem involving a relationship between two numbers.

14. Positive Numbers The product of two positive numbers is 245. Determine the two numbers if one number is 5 times the other.

1) What are we trying to find?
Find Both Numbers

2) Identify 2 Parts
\[ x = 1st \# \]
\[ 5x = 2nd \# \]

3) Equation:

Product is 245
\[ x(5x) = 245 \]

\[ 5x^2 = 245 \]
\[ \frac{5x^2}{5} = \frac{245}{5} \]
\[ \sqrt{x^2} = \sqrt{49} \]
\[ x = \pm 7 \]

Alternative

\[ x^2 = 49 \]

 Difference of 2 Squares
\[ x^2 - 9 = 0 \]
\[ (x + ?)(x - 7) = 0 \]
\[ x + 7 = 0 \text{ or } x - 7 = 0 \]
\[ x = -7 \text{ or } x = 7 \]
18. Consecutive Odd Integers The product of two consecutive positive odd integers is 143. Determine the two integers.

1) Find 2 Consecutive Odd Positive Integers
2) \( x = 1 \text{st odd} \)
   \( x + 2 = \text{next odd} \)
3) Equation
   \[
   \text{Product} = 143
   \]
   \[
   x(x+2) = 143
   \]
   \[
   x^2 + 2x = 143
   \]
   \[
   x^2 + 2x - 143 = 0
   \]
   \[
   (x+13)(x-11) = 0
   \]
   \[
   x+13 = 0 \text{ or } x-11 = 0
   \]
   \[
   x = -13 \text{ or } x = 11
   \]
   \[
   x = 11 \text{ or } x = 13
   \]
   \[
   x+2 = 13
   \]
   \[
   x = 11 \text{ or } x+2 = 13
   \]
20. **Rectangular Scrapbook** A scrapbook page has an area of 180 square inches. Find the length and width if the width is 3 inches less than the length.

\[ \text{Find } \text{Length} \times \text{Width} \]
\[ L = \text{Length} \]
\[ W = \text{Width} \]
\[ W + 3 = \text{Length} \]

\[ \text{Area} = \text{Length} \times \text{Width} \]
\[ 180 = W(W + 3) \]
\[ 180 = W^2 + 3W \]

Factoring:
\[ 0 = W^2 + 3W - 180 \]
\[ 0 = (W - 12)(W + 15) \]

\[ \text{Width} = 12 \]
\[ \text{Length} = 15 \]

\[ W - 12 = 0 \text{ or } W + 15 = 0 \]
\[ W = 12 \text{ or } W = -15 \]
EXAMPLE 3  Earth's Gravitational Field In earth's gravitational field, the distance, \( d \), in feet, that an object falls \( t \) seconds after it has been released is given by the formula \( d = 16t^2 \). While at the top of a roller coaster, a rider's eyeglasses slide off his head and fall out of the cart. How long does it take the eyeglasses to reach the ground 64 feet below?

**Solution** Understand and Translate Substitute 64 for \( d \) in the formula and then solve for \( t \).

\[
\begin{align*}
4 & = 16t^2 \\
64 & = 16t^2 \\
\frac{64}{16} & = t^2 \\
4 & = t^2
\end{align*}
\]

**Carry Out**

Now subtract 4 from both sides of the equation and write the equation with 0 on the right side to put the quadratic equation in standard form.

\[
\begin{align*}
4 - 4 & = t^2 - 4 \\
0 & = t^2 - 4 \\
(t + 2)(t - 2) & = 0
\end{align*}
\]

or \( t^2 - 4 = 0 \)

\[
\begin{align*}
t + 2 & = 0 & \text{or} & & t - 2 & = 0 \\
t & = -2 & & \text{or} & & t & = 2
\end{align*}
\]

**Check and Answer** Since \( t \) represents the number of seconds, it must be a positive number. Thus, the only possible answer is 2 seconds. It takes 2 seconds for the eyeglasses (or any other object falling under the influence of gravity) to fall 64 feet.

26. Falling Rock How long would it take a rock that falls from a cliff 400 feet above the sea to hit the sea?
Learn The Pythagorean Theorem

Now we will introduce the Pythagorean Theorem, which describes an important relationship between the length of the sides of a right triangle. The Pythagorean Theorem is named after Pythagoras of Samos (≈569 B.C.–475 B.C.) who was born in Samos, Ionia. Pythagoras is often described as the first pure mathematician. Unlike many later Greek mathematicians, relatively little is known about his life. The society he led, the Pythagorians, was half religious and half scientific. They followed a code of secrecy and did not publish any of their writings. There is fairly good agreement on the main events of Pythagoras’s life, but many of the dates are disputed by scholars. Now let’s discuss the Pythagorean Theorem.

A right triangle is a triangle that contains a right, or 90°, angle (Fig. 5.4). The two shorter sides of a right triangle are called the legs and the largest side, which is always opposite the right angle is called the hypotenuse. The Pythagorean Theorem expresses the relationship between the lengths of the legs of a right triangle and its hypotenuse.

Pythagorean Theorem

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.

\[(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2\]

If \(a\) and \(b\) represent the legs, and \(c\) represents the hypotenuse, then

\[a^2 + b^2 = c^2\]

When you use the Pythagorean Theorem, it makes no difference which leg you designate as \(a\) and which leg you designate as \(b\), but the hypotenuse is always designated as \(c\).

Helpful Hint

When drawing a right triangle, the hypotenuse, \(c\), is always the side opposite the right angle. See Figures 5.6 (a)–(d).

Notice that the hypotenuse is always the longest side of a right triangle.
28. \( a = 16, c = 20, b = 22 \)
6.  \[ 8^2 + 15^2 = c^2 \]
\[ 64 + 225 = c^2 \]
\[ \sqrt{289} = \sqrt{c^2} \]
\[ 17 = c \]

8.  \[ a^2 + 24^2 = 26^2 \]
\[ a^2 + 576 = 676 \]
\[ \sqrt{a^2} = \sqrt{100} \]
\[ a = 10 \]
38. Laptop The top of a new experimental rectangular laptop computer has a diagonal of 17 inches. If the length of the computer is 1 inch less than twice its width, find the dimensions of the computer.