### 7.3 Slope of a Line

1. Find the slope of a line.
2. Recognize positive and negative slopes.
3. Examine the slopes of horizontal and vertical lines.
4. Examine the slopes of parallel and perpendicular lines.

#### Find the Slope of a Line

In this section we discuss the **slope** of a line. In the following Helpful Hint we discuss similarities between slope as commonly used and the slope of a line.

**Helpful Hint** Slope

We often come across slopes in everyday life. A highway (or a ramp) may have a grade (or slope) of 8%. A roof may have a pitch (or slope) of $\frac{6}{15}$. The slope is a measure of steepness which can be determined by dividing the vertical change, called the rise, by the horizontal change, called the run.

Suppose a road has an 8% grade. Since $8\% = \frac{8}{100}$, this means the road drops (or rises) 8 feet for each 100 feet of horizontal length. A roof pitch of $\frac{6}{15}$ means the roof drops 6 feet for each 15 feet of horizontal length.

When we find the slope of a line we are also finding a ratio of the vertical change to the horizontal change. The major difference is that when we find the slope of a non-horizontal, non-vertical line, the slope can be a positive number or a negative number, as will be explained shortly.

The slope of a line is a measure of the steepness of the line. The slope of a line is an important concept in many areas of mathematics. A knowledge of slope is helpful in understanding linear equations. We now define the slope of a line.

**Slope of a Line**

The slope of a line is a ratio of the vertical change to the horizontal change between any two selected points on the line.

$$m = \frac{\text{Vertical Change}}{\text{Horizontal Change}} = \frac{\text{rise}}{\text{run}}$$
slope = \frac{\text{vertical change}}{\text{horizontal change}}

\begin{align*}
(5,7) & \quad m = \frac{7-2}{5-1} = \frac{5}{4} \\
(4,2) & \\
(-8,-2) & \quad m = \frac{-2-(-6)}{-8-(-3)} = \frac{4}{5} \\
(-3,6) & \quad m = \frac{6-(-5)}{-3-(-5)} = \frac{11}{2}
\end{align*}
Now we present the procedure to find the slope of a line between any two points \((x_1, y_1)\) and \((x_2, y_2)\). Consider Figure 7.34.

The vertical change can be found by subtracting \(y_1\) from \(y_2\). The horizontal change can be found by subtracting \(x_1\) from \(x_2\).

**Figure 7.34**

**Slope of a Line through the Points \((x_1, y_1)\) and \((x_2, y_2)\)**

\[
\text{Slope} = \frac{\text{change in } y \text{ (vertical change)}}{\text{change in } x \text{ (horizontal change)}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

14. \((-4, 2)\) and \((6, 5)\)

\[
m = \frac{5 - 2}{6 - (-4)} = \frac{3}{10}
\]

20. \((-7, 8)\) and \((3, -1)\)

\[
m = \frac{8 - (-1)}{-7 - 3} = \frac{9}{-10}
\]
2 Recognize Positive and Negative Slopes

A straight line for which the value of y increases as x increases has a **positive slope**; see Figure 7.35a. A line with a positive slope rises as it moves from left to right. A straight line for which the value of y decreases as x increases has a **negative slope**; see Figure 7.35b. A line with a negative slope falls as it moves from left to right.

![Figure 7.35](image_url)

**FIGURE 7.35**

(a) Line rises from left to right

(b) Line falls from left to right

26. **Positive Slope**

28. **Negative Slope**
In Exercises 71 and 72, find the slope of the line segments indicated in a) red and b) blue.

72. **Stopping Distance on Wet Pavement for Midsize Car**

![Graph showing stopping distance vs. speed for a midsize car on wet pavement.](image)

Source: Automobile Association of America
3 Examine the Slopes of Horizontal and Vertical Lines

Now we consider the slope of horizontal and vertical lines.

Consider the graph of \( y = 5 \) (Fig. 7.39). What is its slope?

The graph is parallel to the x-axis and goes through the points (2, 5) and (6, 5). Arbitrarily select (6, 5) as \((x_2, y_2)\) and (2, 5) as \((x_1, y_1)\). Then the slope of the line is

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{6 - 2} = \frac{0}{4} = 0
\]

Since there is no change in \( y \), this line has a slope of 0. Note that any two points on the line would yield the same slope, 0.

**Slope of a Horizontal Line**

Every horizontal line has a slope of 0.

Now we discuss vertical lines. Consider the graph of \( x = 2 \) (Fig. 7.40). What is its slope?

The graph is parallel to the y-axis and goes through the points (2, 1) and (2, 4). Arbitrarily select (2, 4) as \((x_2, y_2)\) and (2, 1) as \((x_1, y_1)\). Then the slope of the line is

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{2 - 2} = \frac{3}{0}
\]

We learned in Section 1.8 that \(0\) is undefined. Thus, we say that the slope of this line is undefined.

**Slope of a Vertical Line**

The slope of any vertical line is undefined.
4. Examine the Slopes of Parallel and Perpendicular Lines

Two lines are parallel when they do not intersect, no matter how far they are extended. Figure 7.41 on page 447 illustrates two parallel lines.

If we compute the slope of line 1 using the given points, we obtain a slope of 3. If we compute the slope of line 2, we obtain a slope of 3. (You should compute the slopes of both lines now to verify this.) Notice both lines have the same slopes. Any two nonvertical lines that have the same slope are parallel lines.

![Figure 7.41](image)

**Parallel Lines**

Two nonvertical lines with the same slope and different y-intercepts are parallel lines. Any two vertical lines are parallel to each other.

66. The slope of a given line is $-2$. If a line is to be drawn parallel to the given line, what will be its slope?
Now let's consider perpendicular lines. Two lines are **perpendicular** when they meet and form a right (90°) angle. **Figure 7.43** illustrates two perpendicular lines.

If we compute the slope of line 1 using the given points, we obtain a slope of $\frac{1}{2}$. If we compute the slope of line 2 using the given points, we obtain a slope of $-2$.

(You should compute the slopes of both lines now to verify this.) Notice the product of their slopes, $\frac{1}{2} \cdot (-2)$, is $-1$. Any two numbers whose product is $-1$ are said to be **negative reciprocals** of each other. In general, if $m$ represents a number, its negative reciprocal will be $-\frac{1}{m}$ because $m \left( -\frac{1}{m} \right) = -1$. Any two lines with slopes that are negative reciprocals of each other are perpendicular lines.

**Perpendicular Lines**

Two lines whose slopes are negative reciprocals of each other are perpendicular lines. Any vertical line is perpendicular to any horizontal line.

68. The slope of a given line is $5$. If a line is to be drawn perpendicular to the given line, what will be its slope?
In Exercises 49–64, $m_1$ represents the slope of line 1, and $m_2$ represents the slope of the distinct line, line 2. Indicate whether line 1 and line 2 are parallel, perpendicular, or neither.

49. $m_1 = -3, m_2 = -3$ (Parallel)

50. $m_1 = \frac{1}{2}, m_2 = -6$ (Neither)

51. $m_1 = -\frac{1}{4}, m_2 = -4$ (Perpendicular)

52. $m_1 = -1, m_2 = -1$ (Same slope)

53. $m_1 = \frac{2}{3}, m_2 = -\frac{3}{2}$ (Perpendicular)

54. $m_1 = 7, m_2 = -7$ (Neither)