7.5 Graphing Linear Inequalities

Graph linear inequalities in two variables.

Graph Linear Inequalities in Two Variables

A linear inequality occurs when the equal sign in a linear equation is replaced with an inequality sign.

Examples of Linear Inequalities in Two Variables

\[3x + 2y > 4\]
\[-x + 3y < -2\]
\[-x + 4y \geq 3\]
\[4x - y \leq 4\]

To Graph a Linear Inequality in Two Variables

1. Replace the inequality symbol with an equal sign.
2. Draw the graph of the equation in step 1. If the original inequality contained the symbol \(=\) or \(=\), draw the graph using a solid line. If the original inequality contained the symbol \(>\) or \(<\), draw the graph using a dashed line.
3. Select any point not on the line and determine whether this point is a solution to the original inequality. If the selected point is a solution, shade the region on the side of the line containing this point. If the selected point does not satisfy the inequality, shade the region on the side of the line not containing this point.

12. \(y < 2x + 1\)
Quiz 7.3 to 7.5

Find the slope of the line containing the given pair of points. If the slope is undefined, state this.

(7, -3) and (4, -6)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - (-3)}{4 - 7} = \frac{-3}{-3} = 1 \]
Find the slope of the line containing the following two points: \( \left( \frac{7}{6}, -3 \right) \) and \( \left( -\frac{1}{3}, -\frac{1}{3} \right) \).

\[
m = \frac{-\frac{1}{3} - (-3)}{-\frac{1}{3} - \frac{2}{6}} = \frac{-\frac{1}{3} + \frac{3}{1}}{-\frac{1}{3} - \frac{2}{6}} = \frac{-\frac{1}{3} + \frac{9}{3}}{-\frac{2}{6} - \frac{2}{6}} = \frac{\frac{8}{3}}{-\frac{4}{6}} = \frac{8}{3} \cdot \frac{3}{2} = \frac{-16}{9}
\]
\[ m = -\frac{6}{4} = -\frac{3}{2} \]

\[ m = \frac{16}{-4} = -\frac{3}{2} \]
Graph the line through (1, -1) with \( m = 2 \).

\[
m = \frac{2}{1} = \frac{-2}{-1}
\]
\( m_1 = \frac{4}{3}, m_2 = -\frac{3}{4} \)

\( M_1 = \frac{4}{3}, M_2 = -\frac{3}{4} \)

\( \text{Perpendicular} \)

\( \text{Perpendicular Line} \)

\( m_1 = \frac{a}{b}, m_2 = -\frac{b}{a} \)

\( \text{Negative Reciprocals} \)
Write the following equation in slope-intercept form and identify the slope and y-intercept.

\[2x + 3y = 11\]

**Standard or General Form**

\[2x + 3y = 11\]

**Slope-Intercept Form**

\[
\frac{3y}{3} = \frac{-2x + 11}{3}
\]

\[y = \frac{-2}{3}x + \frac{11}{3}\]

**Slope-Intercept Form**

\[
M = \frac{-2}{3}
\]

**Y-intercept**

\((0, \frac{11}{3})\)
\[ M = \frac{-y}{x} \]

\[ y = mx + b \]

\[ y = -\frac{4}{5}x + 5 \]

Equation
Determine whether the given pair of lines are parallel, perpendicular, or neither.

\[4x + 3y = 7\]
\[8x = -6y + 7\]

\[
\begin{align*}
8x &= -6y + 7 \\
-8x &= -8x \\
0 &= -6y - 8x + 7 \\
 6y &= -8x + 7 \\
 y &= -\frac{8}{6}x + \frac{7}{6} \\
 y &= -\frac{4}{3}x + \frac{7}{6}
\end{align*}
\]

\[
\begin{align*}
4x + 3y &= 7 \\
-4x &= -4x \\
3y &= -4x + 7 \\
\frac{3y}{3} &= \frac{-4x + 7}{3} \\
y &= -\frac{4}{3}x + \frac{7}{3}
\end{align*}
\]

m = \frac{-4}{3}

(0, \frac{7}{3})

m = \frac{-4}{3}

Parallel Lines
Same Slope
Write the equation of the line, with the given properties, in slope-intercept form.

Through \((-5, -3)\) and \((-3, 5)\)

Find Equation

\[ m = \text{Slope} = \frac{-3 - (-5)}{-5 - (-3)} \]
\[ m = \frac{-3}{-2} = \frac{3}{2} \]

Point-Slope
\[ y - y_1 = m(x - x_1) \]
\[ y - 5 = \frac{3}{2}(x + 3) \]
\[ y - 5 = \frac{3}{2}x + \frac{9}{2} \]
\[ y = \frac{3}{2}x + 12 + 5 \]
\[ y = \frac{3}{2}x + 17 \]

Graph

\( \text{(6, 12)} \)

\( m = 4 \)
\[ ax + by = c \]

**Standard Form Linear Equation**

**Slope:**

\[ M = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \]

**Horizontal Lines**

Zero Slope

\[ y = \# \]

**Vertical Lines**

Undefined Slope

\[ x = \# \]

\[ y = mx + b \]

Use with Slope

**Slope-Intercept**

\[ m = \text{Slope} \]

\[ y = \text{Intercept} (0, b) \rightarrow y = \text{Intercept} \]

\[ y - y_1 = m (x - x_1) \]

Use with Point & Slope

Point Slope

\[ m = \text{Slope} (x_1, y_1) \rightarrow \text{Slope} \]

Use with 2 Points

Use with 2 Points
Graph the inequality.

\[ x \leq -\frac{9}{2} \]

\[ x = -\frac{9}{2} = -4.5 \]

Solid Boundary Line \( x = -4.5 \)

Vertical Line Undefined Slope

\[ x \leq -\frac{9}{2} \]

Shade left
Graph the inequality.

\[ y < x - 1 \]

\[ y = x - 1 \]

\[ m = \frac{1}{1} = 1 \]

\[ (0, -1) \]

\[ y \text{-intercept} \]

\[ y \leq x - 1 \]

Shade below

Dashed Boundary Line

Test Point:

\( (0,0) \)

\( 0 < 0 - 1 \)

False
\[ 3x - 5y \leq 15 \]

Divide by negative and flip sign:

\[ -5y \leq -3x + 15 \]

\[ y \geq \frac{3}{5}x - 3 \]

Shade above.

Equation of boundary line:

\[ y = \frac{3}{5}x - 3 \]

Test point \((0,0)\):

\[ 0 \geq \frac{3}{5}(0) - 3 \]

False

Test point \((6,0)\):

\[ 0 \geq \frac{3}{5}(6) - 3 \]

True
Graph the inequality.

\[ 7y > 6x - 7 \]

\[ y > \frac{6}{7} x - \frac{1}{7} \]

Point: \((0, -1)\)

Slope: \(m = \frac{6}{7}\)
Chapter 7 Review

(5,8) Undefined Slope

Vertical Line

x = 5

(5,8) Zero Slope

Horizontal Line

y = 8
\[ x + 3y = 65 \]
\[ 3x + 3y = 15 \]
\[ 3y = 12 \]
\[ y = 4 \]
\[ x = 3 \]
\[ y = 4 \]
\[ x = 0 \]
\[ y = 5 \]
\[ \frac{1}{4}x + 2 = 2y \]

\[ x = 0 \]
\[ 2 = 2y \]
\[ 1 = y \]
\[ y = 0 \]

\[ \frac{1}{4}x + 2 = 0 \]
\[ \frac{1}{4}x = -2 \]
\[ x = 8 \]
12. a) \( C(x) = 15 + 0.20x \)
   \( C(20) = 0.20(20) + 15 \)
   \[ = 4 + 15 \]
   \[ = 19 \]

b) 2nd Graph

c) \( C = 0.20x + 15 \)
\[
\begin{align*}
20 & = 0.20x + 15 \\
-15 & = -15
\end{align*}
\]
\[ S = 0.20x \]
\[ 25 = x \]
\[ M = \frac{7}{8} \quad (2) - 1 \]

Point Form

\[ y - y_1 = m (x - x_1) \]

\[ y - 1 = \frac{7}{8} (x - 2) \]

\[ y + 1 = \frac{7}{8} x - \frac{49}{8} \]

\[ \frac{7}{8} \cdot \frac{2}{1} = -\frac{8}{8} \]

Standard Form

\[ \frac{8}{1} \left[ y = \frac{7}{8} x - \frac{47}{8} \right] \]

[\( y = \frac{7}{8} x - \frac{47}{8} \)]

Steps, Intercept Form

\[ 8y = 7x - 47 \]

\[ -7x \]

\[ -7x \]

\[ -1 \left[ -7x + 8y = -47 \right] - 1 \]

\[ -7x - 8y = 47 \]
16. \( y > -\frac{x}{2} + 2 \)
24. $4x - 2y \leq 6$
26. Determine whether \((-3, 7)\) is a solution to each inequality.
   a) \(-2x + 3y < 9\)  
   b) \(-2x + 3y > 9\)  
   c) \(-2x + 3y \geq 9\)  
   d) \(-2x + 3y \leq 9\)
$3x - 2y \leq -12$