1.8 Multiplication and Division of Real Numbers

1. Multiply numbers.
2. Divide numbers.
3. Remove negative signs from denominators.
4. Evaluate divisions involving 0.

1 Multiply Numbers

The following rules are used in determining the sign of the product when two numbers are multiplied.

<table>
<thead>
<tr>
<th>The Sign of the Product of Two Real Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The product of two numbers with like signs is a <strong>positive</strong> number.</td>
</tr>
<tr>
<td>2. The product of two numbers with unlike signs is a <strong>negative</strong> number.</td>
</tr>
</tbody>
</table>

The product of two positive numbers or two negative numbers will be a positive number. The product of a positive number and a negative number will be a negative number.

**EXAMPLE 1** Evaluate.  

a) \(4(-5)\)  
\[-20\]

b) \((-6)(7)\)  
\[-42\]

c) \((-9)(-3)\)  
\[+27\]

**EXAMPLE 2** Evaluate.  

a) \((-8)(5)\)  
\[-40\]

b) \((-4)(-8)\)  
\[32\]

c) \(0(6)\)  
\[0\]

d) \(0(-2)\)  
\[0\]
EXAMPLE 4 Evaluate.

a) \((-5)(-3)(1)(-4)\)

\[\begin{align*}
15 \times (1) &= 15 \\
15 \times (-4) &= -60 \\
3 \text{ Negatives} &\quad \text{Odd} \\
\text{Answer must} &\quad \text{be Negative}
\end{align*}\]

b) \((-2)(-4)(-1)(3)(-4)\)

\[\begin{align*}
8 \times (-1) &= -8 \\
-8 \times (3) &= -24 \\
4 \text{ Negatives} &\quad \text{Even} \\
\text{Answer must} &\quad \text{be Positive}
\end{align*}\]
2 Divide Numbers

The rules for dividing numbers are very similar to those used in multiplying numbers.

**The Sign of the Quotient of Two Real Numbers**

1. The quotient of two numbers with **like** signs is a **positive** number.
2. The quotient of two numbers with **unlike** signs is a **negative** number.

**EXAMPLE 5** Evaluate.

\[
\begin{align*}
\text{a)} & \quad \frac{10}{-5} \quad \text{b)} & \quad \frac{-45}{5} \quad \text{c)} & \quad \frac{-36}{-6} \\
& \quad -2 \quad \quad & \quad -9 \quad \quad & \quad 6
\end{align*}
\]

**EXAMPLE 6** Evaluate.

\[
\begin{align*}
\text{a)} & \quad -16 \div (-2) \quad \text{b)} & \quad \frac{-2}{3} \div \frac{-5}{7} = \frac{-2}{3} \times \frac{-7}{3} \\
& \quad \frac{-16}{-2} = 8 \quad \quad & \quad = \frac{14}{15}
\end{align*}
\]

\[
\begin{align*}
\text{Invert} & \quad \text{& Multiply}
\end{align*}
\]
EXAMPLE 7 Evaluate, rounding your answer to the nearest hundredth when appropriate.

\[ \text{a) } -18.86 \div 4.1 \quad \text{b) } \frac{-27.2}{-2.6} \]

**Helpful Hint**

For multiplication and division of two real numbers:

\[
\begin{align*}
(+)(+) &= + \\
(-)(-) &= + \\
(+)(-) &= - \\
(-)(+) &= - \\
\end{align*}
\]

Like signs give positive products and quotients.

Unlike signs give negative products and quotients.
The Quotient of a Positive Number and a Negative Number

If $a$ and $b$ represent any real numbers, $b \neq 0$, then

\[
\frac{a}{-b} = \frac{-a}{b} = \frac{-a}{b}
\]

\[
\frac{5}{-7}
\]

should be written as either $\frac{-5}{7}$ or $-\frac{5}{7}$.

\[
\frac{10}{-5} = -2 \quad - \quad \frac{-10}{5} = -2
\]
Evaluate Divisions Involving 0

What is $\frac{0}{1}$ equal to? Note that $\frac{6}{3} = 2$ because $3 \cdot 2 = 6$. We can follow the same procedure to determine the value of $\frac{0}{1}$. Suppose that $\frac{0}{1}$ is equal to some number, which we will designate by $\square$.

If $\frac{0}{1} = \square$ then $1 \cdot \square = 0$

Since only $1 \cdot 0 = 0$, the $\square$ must be 0. Thus, $\frac{0}{1} = 0$. Using the same technique, we can show that zero divided by any nonzero number is zero.

### Zero Divided by a Nonzero Number

If $a$ represents any real number except 0, then

\[
\frac{0}{a} = \frac{0 \cdot a}{a} = 0
\]

Now what is $\frac{1}{0}$ equal to?

If $\frac{1}{0} = \square$ then $0 \cdot \square = 1$

But since 0 multiplied by any number will be 0, there is no value that can replace $\square$. We say that $\frac{1}{0}$ is undefined. Using the same technique, we can show that any real number, except 0, divided by 0 is undefined.

### Division by Zero

If $a$ represents any real number except 0, then

\[
a \div 0 \quad \text{or} \quad \frac{a}{0}
\]

is undefined.

\[
\frac{6}{0} = \text{Undefined}
\]

\[
\frac{6}{2} = 3 \quad 2 \times 3 = 6
\]

\[
\frac{0}{2} = 0 \quad 2 \times 0 = 0
\]

\[
\frac{6}{0} = ? \quad ? \times 0 = 6
\]

No Answer
EXAMPLE 9 Indicate whether each quotient is 0 or undefined.

a) \( \frac{0}{2} \)  

b) \( \frac{5}{0} \)  

c) \( \frac{0}{-4} \)  

d) \( \frac{-2}{0} \)
20. $6(-2)$
24. $-1(8.7)$
28. $(7)(-8)$
32. $5(-4)(2)$
36. $(2)(-4)(-5)(-1)$
40. $(-6)(6)(4)(-4)$

44. $\left(\frac{4}{5}\right)\left(\frac{-3}{10}\right)$
48. $\left(\frac{-9}{10}\right)\left(\frac{7}{-8}\right)$
52. \( \frac{-18}{9} \)
56. \( \frac{-15}{-1} \)
60. \( \frac{63}{-7} \)
64. \( \frac{-10}{10} \)

68. Divide 26 by \(-13\).
72. Divide \(-36\) by \(-6\).

76. \( \frac{6}{15} \div \left( \frac{7}{30} \right) \)
80. \( \frac{-16}{3} \div \left( \frac{5}{-9} \right) \)

92. \((6)(1)(-3)(4)\)

94. \( \frac{-2.7}{0} \)

96. \( \frac{0}{1} \)