2.7 Ratios and Proportions

1. Understand ratios.
2. Solve proportions using cross-multiplication.
3. Solve applications.
4. Use proportions to change units.
5. Use proportions to solve problems involving similar figures.

1. Understand Ratios

A ratio is a quotient of two quantities. Ratios provide a way to compare two numbers or quantities. The ratio of the number \( a \) to the number \( b \) may be written

\[
\frac{a}{b}, \quad a:b, \quad \text{or} \quad \frac{a}{b}
\]

where \( a \) and \( b \) are called the terms of the ratio. Notice that the symbol : can be used to indicate a ratio.

Practice the Skills

The results of a mathematics examination are 6 A's, 4 B's, 9 C's, 3 D's, and 2 F's. Write the following ratios in lowest terms.

14. F's to total grades
16. Grades better than C to total grades
18. Grades better than C to grades less than C

20. 50 dollars to 60 dollars
22. 18 liters to 24 liters
24. 6 feet to 4 yards
26. 26 ounces to 4 pounds

In Exercises 27 and 28, find the gear ratio. Write the ratio as some quantity to 1. (See Example 4.)

28. Driving gear, 30 teeth; driven gear, 8 teeth
2 Solve Proportions Using Cross-Multiplication

A proportion is a special type of equation. It is a statement of equality between two ratios. One way of denoting a proportion is \(a:b = c:d\), which is read “\(a\) is to \(b\) as \(c\) is to \(d\).” In this text we write proportions as

\[
\frac{a}{b} = \frac{c}{d}
\]

The \(a\) and \(d\) are referred to as the extremes, and the \(b\) and \(c\) are referred to as the means of the proportion. In Sections 2.4 and 2.5, we solved equations containing fractions by multiplying both sides of the equation by the LCD to eliminate fractions. For example, for the proportion

\[
\frac{\frac{3}{4}}{\frac{4}{9}} = \frac{3 \times 9}{4 \times 4} = \frac{27}{16}
\]

Note that the product of the extremes is equal to the product of the means.

If any three of the four quantities of a proportion are known, the fourth quantity can easily be found.

**EXAMPLE 5** Solve \(\frac{x}{3} = \frac{35}{15}\) for \(x\) by cross-multiplying.

**Solution**

\[
x \times 3 = \frac{35}{15}
\]

\[
x \cdot 15 = 3 \cdot 35
\]

\[
15x = 105
\]

\[
x = \frac{105}{15} = 7
\]

**Check**

\[
\frac{x}{3} = \frac{35}{15}
\]

\[
\frac{7}{3} = \frac{35}{15}
\]

\[
\frac{7}{3} = \frac{7}{3}
\]

True

38. \(\frac{x}{8} = \frac{24}{48}\)

42. \(\frac{-12}{13} = \frac{36}{x}\)

46. \(\frac{3}{12} = \frac{-1.4}{x}\)

\[
\frac{-12}{13} = \frac{36}{x}
\]

\[
-12x = 13 \times 36
\]

\[
-12x = 468
\]

\[
-12x = \frac{468}{-12}
\]

\[
x = -39
\]

\[
\frac{4}{8} = \frac{24}{48}
\]

\[
\frac{48 \cdot x = 8 \cdot 24}{48}
\]

\[
\frac{48x = 192}{48}
\]

\[
x = 4
\]

3
3 Solve Applications

Often, practical problems can be solved using proportions. To solve such problems, use the five-step problem-solving procedure we have been using throughout the book. Below we give that procedure with more specific directions for translating problems into proportions.

To Solve Problems Using Proportions

1. Understand the problem.
2. Translate the problem into mathematical language.
   a) First, represent the unknown quantity by a variable (a letter).
   b) Second, set up the proportion by listing the given ratio on the left side of the equal sign, and the unknown and the other given quantity on the right side of the equal sign. When setting up the right side of the proportion, the same respective quantities should occupy the same respective positions on the left and the right. For example, an acceptable proportion might be

   \[
   \text{Given ratio} \quad \frac{\text{miles}}{\text{hour}} = \frac{\text{miles}}{\text{hour}}
   \]

3. Carry out the mathematical calculations necessary to solve the problem.
   a. Once the proportion is correctly written, drop the units and cross-multiply.
   b. Solve the resulting equation.
4. Check the answer obtained in step 3.
5. Make sure you have answered the question.

Note that the two ratios used in a proportion must have the same units. For example, if one ratio is given in miles/hour and the second ratio is given in feet/hour, one of the ratios must be changed before setting up the proportion.

62. Spreading Fertilizer If a 40-pound bag of fertilizer covers 5000 square feet, how many pounds of fertilizer are needed to cover an area of 26,000 square feet?

\[
\frac{40 \text{ lb.}}{5000 \text{ ft}^2} = \frac{x \text{ lb.}}{26,000 \text{ ft}^2}
\]
56. Laying Cable A telephone cable crew is laying cable at a rate of 42 feet an hour. How long will it take them to lay 252 feet of cable?

\[
\frac{42 \text{ ft.}}{1 \text{ hr.}} = \frac{252 \text{ ft.}}{x \text{ hr.}}.
\]

70. Dosage by Body Surface A doctor asks a nurse to administer 0.7 gram of meprobamate per square meter of body surface. The patient's body surface is 0.6 square meter. How much meprobamate should be given?

\[
\frac{0.7 \text{ grams}}{1 \text{ m}^2} = \frac{x \text{ grams}}{0.6 \text{ m}^2}.
\]
4 Use Proportions to Change Units

Proportions can also be used to convert from one quantity to another. For example, you can use a proportion to convert a measurement in feet to a measurement in meters, or to convert from pounds to kilograms. The following examples illustrate converting units.

EXAMPLE 10 Kilometers to Miles There are approximately 1.6 kilometers in 1 mile. What is the distance, in miles, of 78 kilometers?

Solution Understand and Translate We know that 1 mile ≈ 1.6 kilometers. We use this known fact in one ratio of our proportion. In the second ratio, we set the quantities with the same units in the same respective positions. The unknown quantity is the number of miles, which we will call x.

\[
\text{Known ratio} \left\{ \frac{1 \text{ mile}}{1.6 \text{ kilometers}} = \frac{x \text{ miles}}{78 \text{ kilometers}} \right. 
\]

Note that both numerators contain the same units, and both denominators contain the same units.

76. Convert 22,704 feet to miles (5280 feet = 1 mile).

\[
\frac{22704 \text{ ft}}{x \text{ miles}} = \frac{5280 \text{ ft}}{1 \text{ mile}}
\]

86. Currency Exchange When Mike Weatherbee visited the United States from Canada, he exchanged $13.50 Canadian for $10 U.S. If he exchanges his remaining $600 Canadian for U.S. dollars, how much more in dollars will he receive?
5 Use Proportions to Solve Problems Involving Similar Figures

Proportions can also be used to solve problems in geometry and trigonometry. The following examples illustrate how proportions may be used to solve problems involving similar figures. Two figures are said to be similar when their corresponding angles are equal and their corresponding sides are in proportion. Two similar figures will have the same shape.

**EXAMPLE 12** The figures to the left are similar. Find the length of the side indicated by the x.

**Solution** We set up a proportion of corresponding sides to find the length of side x.

\[
\begin{align*}
\text{Lengths from smaller figure} & \quad \text{Lengths from larger figure} \\
5 \text{ inches and 12 inches are corresponding sides of similar figures.} & \quad \rightarrow \quad \frac{5}{2} = \frac{12}{x} \\
2 \text{ inches and } x \text{ are corresponding sides of similar figures.} & \quad \rightarrow \quad 5x = 24 \\
& \quad x = \frac{24}{5} = 4.8
\end{align*}
\]

Thus, the side indicated by x is 4.8 inches in length.

50. \[
\frac{2 \text{ ft}}{0.8 \text{ ft}} = \frac{1.8 \text{ ft}}{x}
\]

52. \[
\frac{5 \text{ ft}}{7 \text{ ft}} = \frac{8 \text{ ft}}{x}
\]