Focal Chord Length and Arc Length

Find the length of the latus rectum of the parabola given by $y^2 = 4px$ length of the parabola arc intercepted by the latus rectum.

Solution: Because the latus rectum passes through the focus $(0, p)$ and due to the axis, the coordinates of the endpoints are $(a, -p)$ and $(a, p)$. By $x$, the equation of the parabola produces

$$y^2 = 4px$$

So, the endpoints of the latus rectum are $(a, -p)$ and $(2a, p)$, and the equation distance is 4a, as shown in Figure 10.5. In contrast, the length of arc is

$$\int_a^{2a} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

Theorem

One widely used property of a parabola is its reflective property: a surface called reflective if the tangent line at any point on any angle with an incoming ray and the resulting outgoing ray is equal to the incoming ray in the angle of incidence, and the angle outgoing ray is the angle of reflection. One example of a reflector.

Another type of reflective surface is the paraboloid a reflector. A special property of parabolic reflectors is that they allow rays parallel to the axis through the focus of the paraboloid behind the design of the parabolic mirrors used in reflecting all light rays emerging from the focus of a parabolic reflector parallel, as shown in Figure 10.6.

THEOREM 10.2 Reflective Property of a Parabola

Let $P$ be a point on a parabola. The tangent line to the parabola makes equal angles with the following two lines:

1. The line passing through $P$ and the focus
2. The line passing through $P$ parallel to the axis of the parabola

An ellipse is the set of all points $(x, y)$ the sum of whose distances from two distinct fixed points called foci is constant. (See Figure 10.7.) The line through the foci intersects the ellipse at two points, called the vertices. The chord joining the vertices is the major axis, and its midpoint is the center of the ellipse. The chord perpendicular to the major axis at the center is the minor axis of the ellipse. (See Figure 10.8.)

THEOREM 10.3 Standard Equation of an Ellipse

At this point, "ellipse" should be replaced with a circle.
THEOREM 10.3 Standard Equation of a Hyperbola

The standard form of the equation of a hyperbola with center at $(h, k)$ is

\[ \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \]

or

\[ \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1 \]

where $a$ and $b$ are the same as in the ellipse, and $c$ is the distance from the center to a vertex of the hyperbola. The asymptotes are the lines that pass through the center of the hyperbola and the points $(h \pm a, k)$ or $(h, k \pm b)$. The transverse axis is the axis of symmetry of the hyperbola, and the conjugate axis is perpendicular to the transverse axis at the center of the hyperbola.

Example: Given the equation of a hyperbola $x^2 - y^2 = 1$, find the center, vertices, foci, and asymptotes. The center is at $(0, 0)$, the vertices are at $(\pm 1, 0)$, the foci are at $(\pm \sqrt{2}, 0)$, and the asymptotes are $y = \pm x$.