Evaluate \[ \int \left[ \cos \left( \frac{m \pi}{n} \right) \cos \left( \frac{n \pi}{m} \right) \right] \, dx \]

\[ \frac{1}{2} \int \left[ \cos \left( \frac{m \pi}{n} \right) + \cos \left( \frac{n \pi}{m} \right) \right] \, dx \]

\[ \frac{1}{2} \int \frac{1}{2} \cos \left( \frac{m \pi}{n} \right) \, dx + \frac{1}{2} \int \frac{1}{2} \cos \left( \frac{n \pi}{m} \right) \, dx \]

\[ \frac{1}{2} \sin \left( \frac{m \pi}{n} \right) + \frac{1}{2} \sin \left( \frac{n \pi}{m} \right) + C \]

Integrals Involving Sine-Cosine Products with Different Angles

Integrals involving the products of sines and cosines of two different angles noted in many applications. In such instances, you can use the following product-to-sum identities:

\[ \sin a \cos b = \frac{1}{2} \left[ \sin(a + b) - \sin(a - b) \right] \]

\[ \cos a \sin b = \frac{1}{2} \left[ \sin(a + b) + \sin(a - b) \right] \]

\[ \cos a \cos b = \frac{1}{2} \left[ \cos(a + b) + \cos(a - b) \right] \]

Sine-Cosine Relations (9 = 4)

1. Prove trigonometric identities: \[ \frac{\sin x}{\cos x} \]

Section 8.5
Recall from algebra that every polynomial with real coefficients can be factored into linear and irreducible quadratic factors. For instance, the polynomial
\[ x^4 - x^3 - 2x^2 + x + 1 \]
can be written as
\[ (x^2 - 1)(x^2 - x - 1) = (x - 1)(x + 1)(x^2 - 1) \]
where \( x - 1 \) is a linear factor, \( x + 1 \) is a repeated linear factor, and \( x^2 - 1 \) is an irreducible quadratic factor. Using this factorization, you can write the partial fraction decomposition of the rational expression.
\[ \int \frac{x^2 + x + 9}{(x^2 + 9)^2} \, dx \]

\[ \frac{1}{3} \text{arctan}(x) + \frac{1}{2(x^2 + 9)} + C \]

\[ \text{Answer} \]

Section 8.5

Count the limits of integration to solve (Easiest)

Extra credit if you convert back to x to get the same answer.