Section 8.7

\[
\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \frac{0}{0}
\]

Use L'Hôpital's Rule.

\[
\lim_{x \to 0} \frac{2 \cos(2x)}{3 \cos(3x)} = \frac{2(1)}{3(0)} = \frac{2}{3}
\]

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\[
\lim_{x \to 0^+} \frac{1}{x^2} = \infty
\]

\[
\ln y = \ln x^3
\]

\[
\ln y = \ln x^3 + \ln x^3
\]

\[
\ln y = \frac{\ln x^3 + \frac{3}{2} \ln x}{\frac{3}{2} \ln x}
\]

\[
y = 3
\]

\[
\lim_{x \to 0} \frac{\ln x}{x^2} = \infty
\]

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Indeterminate Forms

\[
0 \times \infty
\]

\[
0 + \infty = \infty
\]

\[
- \infty - \infty = - \infty
\]

\[
\infty - \infty = 0
\]

\[
0 \div \infty = 0
\]

\[
0 - 0 = \frac{0}{0}
\]

Oct 7-10:22 AM
Improper Integrals

- Evaluate an improper integral that has an infinite limit of integration.
- Evaluate an improper integral that has an infinite discontinuity.

Improper Integrals with Infinite Limits of Integration

The definition of a definite integral

\[ \int_{a}^{b} f(x) \, dx \]

requires that the interval \([a, b]\) be finite. Furthermore, the Fundamental Theorem of Calculus, by which you have been evaluating definite integrals, makes no mention of \(a\) or \(b\). In this section, you will study procedures for evaluating integrals that do not satisfy these requirements—usually because either one or both of the limits are infinite, or \(f\) has a finite number of infinite discontinuities along \([a, b]\).

Integrals that possess either property are called improper. If \(f\) is said to have an infinite discontinuity at \(x = c\), from the right

\[ \lim_{x \to c^+} f(x) = \pm \infty \]

\[ \lim_{x \to c^-} f(x) = \pm \infty \]

The function \(f\) in this case is called an improper integral.

Definition of Improper Integrals with Infinite Limits

1. If \(f\) is continuous on the interval \([a, b]\), then

\[ \int_{a}^{b} f(x) \, dx = \lim_{t \to b^-} \int_{a}^{t} f(x) \, dx \]

2. If \(f\) is continuous on the interval \((-\infty, b]\), then

\[ \int_{-\infty}^{b} f(x) \, dx = \lim_{t \to -\infty} \int_{t}^{b} f(x) \, dx \]

3. If \(f\) is continuous on the interval \([a, \infty)\), then

\[ \int_{a}^{\infty} f(x) \, dx = \lim_{t \to \infty} \int_{a}^{t} f(x) \, dx \]

4. If \(f\) is continuous on the interval \([-\infty, \infty)\), then

\[ \int_{-\infty}^{\infty} f(x) \, dx = \lim_{t \to \infty} \int_{-t}^{t} f(x) \, dx \]
Definition of Improper Integrals with Infinite Integration Limits

1. If \( f(x) \) is continuous on the interval \([a, b)\), then
   \[
   \int_a^b f(x) \, dx = \lim_{b \to b^-} \int_a^b f(x) \, dx.
   \]

2. If \( f(x) \) is continuous on the interval \((a, b]\), then
   \[
   \int_a^b f(x) \, dx = \lim_{a \to a^+} \int_a^b f(x) \, dx.
   \]

3. If \( f(x) \) is continuous on the interval \((-\infty, b)\), then
   \[
   \int_{-\infty}^b f(x) \, dx = \lim_{a \to -\infty} \int_a^b f(x) \, dx.
   \]

4. If \( f(x) \) is continuous on the interval \((a, \infty)\), then
   \[
   \int_a^{\infty} f(x) \, dx = \lim_{b \to \infty} \int_a^b f(x) \, dx.
   \]

where \( c \) is any real number (see Exercise 11).

In the first two cases, the improper integral converges if the limit exists—otherwise, the improper integral diverges. In the third case, the improper integral on the left diverges if either of the improper integrals on the right diverges.