In Exercises 13–26, find $M_x, M_y$, and $(\bar{x}, \bar{y})$ for the lamina of uniform density $\rho$ bounded by the graphs of the equations.

13. $y = 1, y = 0, x = 2$
14. $y = -x + 3, y = 0, x = 0$
15. $y = \sqrt{x}, y = 0, x = 4$
16. $y = \frac{1}{2}x, y = 0, x = 3$
17. $y = x, y = x^2$
18. $y = \sqrt{2x}, y = 0, x = 1$
19. $y = x^2 + 4x + 3, y = x + 2$

\[ -x^2 + 3x = 0 \]
\[ -x(x-3) = 0 \]
\[ -x^3 = 0 \text{ or } x^3 = 0 \]
\[ x = 0 \text{ or } x = 3 \]

\[ (0, 0) \quad (3, 0) \]

From

\[ m = \rho \int_0^3 \left( (-x^2 + 3x) - (x^2) \right) dx \]
\[ + \rho \int_0^3 (-x^2 + 3x) \ dx \]
\[ = \rho \left[ \left( -\frac{x^3}{3} + \frac{3x^2}{2} \right) \right]_0^3 \]
\[ = \rho \left( -\frac{9}{3} + \frac{27}{2} \right) \]

\[ M_x = \rho \int_0^3 \left( x^2 + 3x \right) \ dx \]
\[ = \rho \left( \frac{3}{2} \right) \]

\[ \bar{x} = \frac{M_x}{m} \]

\[ \bar{y} = \frac{\rho}{m} \int_0^3 \left( x^3 \right) dx \]

\[ \rho = \frac{M_y}{m} \]

\[ \rho = \frac{\frac{3}{2}}{m} \]

\[ \bar{y} = \frac{\rho}{m} \int_0^3 \left( x^3 \right) dx \]

\[ = \frac{3}{2} \]

\[ \bar{y} = \frac{3}{2} \]

Section 2.6

\[ \bar{y} = \frac{3}{2} \]

\[ \bar{x} = \frac{M_x}{m} \]

\[ \bar{y} = \frac{3}{2} \]

Center of Mass $=(\bar{x}, \bar{y})$
23. **Pumping Water**  A hemispherical tank of radius 6 feet is positioned so that its base is circular. How much work is required to fill the tank with water through a hole in the base if the water source is at the base?

**Volume of a Disk:**

\[ V = \pi r^2 h = \text{Volume of Disk} \]

\[ V = \pi (36-y^2) dy = \text{Force} \]

**Weight of Disk**

\[ W = 62.4 \pi (36-y^2) dy \]

**Work**

\[ W = 62.4 \pi \int_0^6 (y)(36-y^2) dy \]

\[ \Delta W = \text{(force increment)(distance)} = (\Delta F)(x). \]

\[ 62.4 \pi \left[ 18y^2 - \frac{y^4}{4} \right]_0^6 \]

\[ 62.4 \pi \left[ 648 - 324 \right] \]

\[ 20,296 \pi \text{ ft-lb} \]
The solutions to Examples 2 and 3 conform to our development of work as the summation of increments in the form

$$\Delta W = \text{(force)} \cdot \text{(distance increment)} = (F)(\Delta x).$$

Another way to formulate the increment of work is

$$\Delta W = \text{(force increment)} \cdot \text{(distance)} = (\Delta F)(x).$$

This second interpretation of $\Delta W$ is useful in problems involving the movement of nonrigid substances such as fluids and chains.
Lifting a Chain  In Exercises 27–30, consider a 20-foot chain that weighs 3 pounds per foot hanging from a winch 20 feet above ground level. Find the work done by the winch in winding up the specified amount of chain.

27. Wind up the entire chain.

\[
\text{Weight of Each Section} = 3 \, \text{dy}
\]

Distance Lifted \((20 - y)\)

\[W = \int_{0}^{20} (20 - y) \, dy\]

\[
\begin{align*}
W &= \int_{0}^{20} (60 - 3y) \, dy \\
&= \left[ 60y - \frac{3y^2}{2} \right]_{0}^{20} \\
&= 1200 - 600 = 600 \text{ ft-lb}
\end{align*}
\]
\[ x^2 + y^2 = a^2 \]

\[ x^2 + x^2 = a^2 \]

\[ 2x^2 = a^2 \]

\[ x^2 = \frac{a^2}{2} \]

\[ x = \frac{a}{\sqrt{2}} \]
\[ A = \pi \int_{a}^{b} f(x) \, dx \]

radius \rightarrow width of rectangles